Essays in Macroeconomics: Information and Financial Markets

by

Luigi Iovino

Submitted to the Department of Economics
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2012

Author ....
Department of Economics
May 15, 2012

Certified by ....
George-Marios Angeletos
Professor of Economics
Thesis Supervisor

Certified by ....
Guido Lorenzoni
Professor of Economics
Thesis Supervisor

Accepted by ....
Michael Greenstone
3M Professor of Environmental Economics
Chairman, Departmental Committee on Graduate Studies

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Abstract

This thesis studies how information imperfections affect financial markets and the macroeconomy. Chapter 1 considers an economy where investors delegate their investment decisions to financial institutions that choose across multiple investment opportunities featuring different levels of idiosyncratic risk and different degrees of correlation with the aggregate of the economy. Investors solve an optimal contracting problem to induce financial institutions to allocate their investment optimally. We then study how investment decisions are affected when financial securities are introduced that allow agents to trade their risks. Investors do not have the necessary information to understand these securities, but give incentives to financial institutions to hedge certain risks. We show that hedging idiosyncratic risks ameliorates the agency problem between investors and financial institutions and reduces aggregate volatility. On the contrary, when aggregate risk can be hedged the agency problem worsens and aggregate volatility increases. Finally, we study the efficiency properties of the equilibrium and the potential role for financial regulation.

Chapter 2 studies the welfare effects of the information contained in macroeconomic statistics, central-bank communications, or news in the media? We address this question in a business-cycle framework that nests the neoclassical core of modern DSGE models. Earlier lessons that were based on "beauty contests" (Morris and Shin, 2002) are found to be inapplicable. Instead, the social value of information is shown to hinge on essentially the same conditions as the optimality of output stabilization policies. More precise information is unambiguously welfare-improving as long as the business cycle is driven primarily by technology and preference shocks—but can be detrimental when shocks to markups and wedges cause sufficient volatility in "output gaps".

Finally, chapter 3 studies how market signals—such as stock prices—can help alleviate the severity of the asymmetric information problem in credit and liquidity management. Asymmetric information hinders the ability of borrowers (firms, investment banks, etc) to undertake profitable investment opportunities and to insure themselves against liquidity shocks. On the equilibrium path, creditors need not learn anything from market signals because they can use a menu of contracts to screen the different types of borrowers. Nevertheless, by conditioning liquidity insurance on ex post price signals, creditors are able to provide the borrowers with better incentives for truth-telling. At the same time, prices depend on the liquidity that creditors offer to the borrowers. This two-way feedback impacts the design of the optimal contract and potentially generates multiple equilibria in financial markets.

Thesis Supervisor: George-Marios Angeletos
Title: Professor of Economics

Thesis Supervisor: Guido Lorenzoni
Title: Professor of Economics

Acknowledgments

This dissertation is the outcome of a long and intense journey. Looking back at the past years, I realize how much less fruitful and enjoyable this journey would have been without the support of so many people around me.

First and foremost, I am extremely grateful to my advisors George Marios Angeletos, Guido Lorenzoni, and Mikhail Golosov for invaluable guidance, support, and encouragement throughout my graduate studies. They provided complementary perspectives on my work, they dedicated substantial time and energy during various stages of my graduate career, and they have all become reference models for my future career.

I also want to thank Daron Acemoglu, Alessandro Bonatti, Bengt Holmström, Leonid Kogan, Bruno Strulovici, Jean Tirole, Iván Werning for helpful conversations, suggestions, and comments. Participants at the MIT macroeconomics seminar also provided very valuable comments on my research.

My experience at MIT could not have been as complete, both from the academic and the human point of view, without the help, advice, and friendship of many of my classmates and friends. In particular, Ruchir Agarwal, Marco Di Maggio, and Stefano Giglio have proved to be invaluable friends. I also want to thank my friends and colleagues Joaquin Blaum, Tom Cunningham, Maya Eden, Lucia Esposito, Patricia Gomez Gonzalez, Felipe Iachan, Marti Mestieri, Plamen Nenov, Michael Peters, Laura Ramos Parra, Giovanni Reggiani, Natalia Rigol, Roberta Scarpato, Annalisa Scognamiglio, Jenny Simon, Alp Simsek, Tom Vogl, Xiao Yu Wang, Anna Zabai.

Finally, and most importantly, I want to thank my family for their love and unwavering support. My mother and father were with me throughout the toughest times. This achievement would have not been possible without their support and sacrifices.

Thank you to all of you.

Ai miei genitori,
a Angela e Piera

Contents

Chapter 1

Sophisticated Intermediation and Aggregate Volatility

1.1 Introduction

The last two decades have witnessed an enormous expansion of markets for financial securities. Derivative instruments, often customized to the specific needs of their users, have become very popular and have enabled firms and financial institutions to manage their risks more efficiently. The over-the-counter (OTC) derivatives market, where contracts are traded directly between two parties (without going through an exchange), has become the largest market for derivatives. The notional amount outstanding of OTC derivatives has increased from 94 trillion dollars in 2000 to 601 trillion dollars by the end of 2010. Commercial banks, in particular, hold large exposures to derivative contracts. The notional value of derivatives held by US commercial banks was estimated to be 244 trillion dollars by the end of 2010. The typical derivatives are interest rate products (such as interest rate swaps) which comprise 82% of the total. Credit derivatives represent 6.1% of the total with their share increasing over time (OCC Quarterly Report, First Quarter 2011). Even though these are notional amounts that can include double counting, it seems that net exposures have increased at a similar pace.

In a perfect world, financial markets allow households and firms to share risks more efficiently. Idiosyncratic risks can be pooled and eliminated with great benefits for risk-averse agents. Aggregate risks – which cannot be eliminated – can be transferred to the agents better equipped to bear them. However, following the dramatic events of recent years, markets for derivatives have come under increased scrutiny. Lack of regulation in derivatives has been partly blamed for the turmoil in financial markets. Many observers have pointed out the "opaqueness" of derivative markets and the "complexity" of the positions held on and off the balance sheets of big financial institutions. Complex financial securities, the argument goes, may actually pose a threat to the system by increasing and concentrating the risks of some institutions.

These observations have motivated a growing literature. Caballero and Simsek (2011) study the possibility that complex financial arrangements may make banks susceptible to contagion; Arora et al. (2011) show how computational complexity amplifies the costs of asymmetric information; Gennaioli et al. (2011) study the possibility that investors neglect certain unlikely events; Farhi and Tirole (2011) show that banks' choices are distorted by the anticipation of collective bail outs; and Stein (2011) studies the "special" demand for riskless assets.

This paper contributes to this literature by focusing on the agency problem between investors and bankers. In particular, I start from the idea that banks perform two types of activities. First, they have the expertise to select projects and monitor their performance. This expertise, however, comes with agency problems: investors have to induce bankers to exert effort and select projects with larger expected returns and lower correlation with the aggregate economy.

Second, banks combine the cash flows of these projects with a rich set of financial securities to hedge risks and produce a final payoff. The way I capture the idea of "complexity" is by assuming that investors do not have the ability to fully understand the banks' balance sheets. This means that in designing the optimal contract investors can only punish and reward the bankers based on their total payoff, i.e., on the sum of the cash flows from real projects and security trading.

The key questions that I address are: (i) why it may be optimal to expose banks to aggregate risk; (ii) whether access to security trading ameliorates or worsens the bankers' incentive problem; (iii) whether there is room for government intervention.

First, I study the benchmark case in which security trading is not allowed. In this case, I show that investors provide incentives by conditioning the payment of the contract on both idiosyncratic and aggregate risks. In particular, they punish financial institutions for generating profits that are very correlated with the rest of the economy, tilting the choice of the bank away from aggregate risk. Even when optimal incentives are in place, however, the agency problem is never fully resolved and, relative to the first best, investors are exposed to excessive aggregate volatility.

Next, I allow banks to trade securities, both securities contingent on idiosyncratic risks and securities contingent on aggregate risk. Investors cannot observe the trading activity of the banks, but can design the optimal contract to influence it. Security trading interacts with the agency problem in different ways. In particular, when banks can trade securities on idiosyncratic risks, this mitigates the agency problem and lowers aggregate volatility. On the contrary, trading of securities on aggregate risk exacerbates the agency problem and increases aggregate volatility. In summary, the effects of the complexity of banks' hedging activity are ambiguous and depend on the relative importance of the two types of risks. Complexity by itself does not necessarily lead to worse economic outcomes and in some cases may reduce volatility in the economy and can be beneficial for investors.

Finally, I derive the normative implications of the interaction between agency problems and security trading. First, I show that, when banks cannot trade financial securities, the equilibrium with the agency problem is constrained efficient. Thus, the higher exposure of investors to aggregate risk relative to the first-best does not by itself open the door to government intervention. When financial securities can be traded, government intervention may be desirable. Inefficiencies originate since investors suffer from a coordination failure. They do not internalize how the activity of trading securities interacts with the agency problem of financial institutions. Therefore, in equilibrium it is too easy for financial institutions to trade securities contingent on aggregate risk. Government intervention can fix this coordination failure and restore efficiency. The government can reduce aggregate volatility (and increase welfare) in different ways. The most effective policy tool is regulation of the issuers of aggregate risk securities. An alternative and less effective policy is to tax transactions in financial markets.

1.2 Related Literature

The core of this paper is a principal-agent model where the principal delegates an investment choice to the agent. The principal provides incentives by exposing the agent to some risk. The seminal contribution of Holmstrom (1979) shows under what conditions more information should be incorporated in the contract. He studies a moral hazard problem with many agents and correlated signals and derives the general principle that observable, correlated signals which are not affected by the agent's effort should not be included in the optimal contract. Contrary to Holmstrom (1979), in this paper the effort of the agents determines the correlation of the projects in the economy and, thus, the optimal contract exposes the agent to the common noise.

The seminal contribution to the literature on delegated portfolio management is Bhattacharya and Pfleiderer (1985) who propose a model where an informed agent has to reveal his information to the principal. The agency problem in this paper arises because managers have access to better information than investors, but they have to be incentivized to collect this information. This is similar to the model of delegated expertise developed by Demski and Sappington (1987) (see also Allen (1990)) and to the delegated portfolio problem with hidden actions (Admati and Pfleiderer (1997), Stoughton (1993)).

The principal-agent model can also be interpreted as a two-tier incentive problem whereby investors lend money to managers who then monitor entrepreneurs who run the projects and choose in what type of risk to invest. The classical paper on delegated management is Diamond (1984) who shows that it is optimal for banks to fully diversify their portfolios. However, Diamond (1984) considers a model where there is no trade-off between different types of risks as in this model.

In the basic version of the model, complexity is modelled by assuming that trades are unobservable (Allen (1985), Arnott and Stiglitz (1993), Hellwig (1983), Bisin and Guaitoli (2004), Cole and Kocherlakota (2001), Bizer and DeMarzo (1999)). Unobservable trades limit risk-sharing in Jacklin

(1987) who shows that financial markets can reduce welfare. Farhi et al. (2009) show how regulation can correct the externality generated by the unobservable trades (see also Allen and Gale (2004) and Golosov (2007)).

In corporate finance, several papers have focused on how hedging opportunities affect incentives when the effort of the managers increase the expected return of the firm (Li (2002), Garvey and Milbourn (2003), Ozerturk (2006), Bisin et al. (2008)). An important difference is Acharya and Bisin (2009) who study a model where firms make investment decisions and can choose the loading on the aggregate state of the economy. They also allow the manager to transfer (aggregate) risk. They focus on the optimal ownership share of the manager and show that a manager who is too risk-averse should own a smaller part of the firm's capital. Acharya and Bisin (2009) do not make the distinction between different types of securities, which is central in this paper, and do not allow investors to write the optimal contract to managers. Also, they study a partial equilibrium model and, thus, they don't consider policy implications[1].

One key difference between the model of this paper and the literature on agency problems with unobservable trades is that in these models agents trade on their own account and undo the incentives provided by the principals. On the contrary, this paper proposes a new way to look at how hidden trades can affect the incentives of the managers. Managers select projects and trade financial securities that affect the balance sheets of the financial institution they manage. Investors observe only the combined payoff of these two activities and, thus, they can provide incentives based only on this payoff. I find this assumption more realistic since complex securities are mostly held on the balance sheets of financial institutions.

A more recent literature studies how the complexity of financial securities and opacity of OTC markets can pose threats to the financial system. Caballero and Simsek (2011) show how complexity (modelled as limited information about the network of counterparties) can potentially cause a cascade of bank failures. Dang et al. (2009) study how some securities, such as debt, that are usually informational insensitive can lose much of their value in bad states of the world because of asymmetric information. Brunnermeier and Oehmke (2011) focus on the definition of complexity when agents are boundedly rational and observe that disclosing more information can lead to *information overload*, which has important implications for designing disclosure requirements and consumer protection. Their reason for regulation is not driven by the agency problem combined with the general equilibrium effects as in this paper.

The paper is organized as follows. Section 1.3 introduces the model and defines the equilibrium. Section 1.4 solves the model for the special case where securities markets are absent. The solution of the model with securities is derived in section 1.5, where I consider the different types of securities separately. In section 1.6, I allow agents to trade both securities and extend the model to include

[1]See Acharya (2009) for a model where firms strategically coordinate their actions and increase the *systemic risk* in the economy.

trading costs. The efficiency properties of the equilibrium are studied in section 1.7 and some optimal policy prescriptions are discussed. Finally, section 1.8 contains the concluding remarks.

1.3 The model

The economy lasts for two periods, $t = 0, 1$ and there is one consumption good. There are two types of agents: investors (the principals) and fund managers (the agents). There is a large number of identical investors. They are born with an endowment of one unit of capital which can be invested to produce consumption goods in period 1. Each investor has access to a continuum of managers, to which he delegates the investment of his capital. The managers of the representative investor form a continuum of measure 1. They are indexed by $i \in [0, 1]$ and have no endowment. They receive capital from the representative investor in period 0 and select projects on his behalf. Investors and managers value consumption only in period 1 according, respectively, to the utility functions $v(\cdot)$ and $u(\cdot)$, which are assumed to be differentiable, increasing and concave.

The economy is characterized by a continuum of sectors, denoted by $j \in [0, 1]$. At time 0, each manager i is randomly matched to a sector j, his area of expertise. The assignment of managers to sectors is one-to-one. Let $\mathcal{F}$ the set of all possible realizations of this matching process. An element $F \in \mathcal{F}$ is a full description of the assignment of managers to sectors.

The investment technology is modelled to capture the idea that managerial effort determines both the expected returns of the selected projects and their correlation with the aggregate economy.

A project requires 1 unit of capital at time 0 and produces a random return at time 1. Each manager i has access to a continuum of potential projects in sector j, some of them are "specialized" projects and have a random payoff

$$r_{i,j} = \bar{r} + \varepsilon_j + u_i$$

some of them are "standard" projects and have a random payoff

$$R_i = \bar{R} + \omega + u_i.$$

$\bar{r}$ and $\bar{R}$ are the expected payoffs of the two projects, ε_j is a sector-specific shock, ω is an aggregate shock, and u_i is a manager-specific shock.

Let K_i denote the units of capital that the manager receives from the investors. After receiving K_i, the manager chooses how much effort to spend in selecting specialized projects. His effort level is denoted by k_i. By exerting effort k_i the manager is able to select exactly k_i specialized projects.[2] A manager exerting effort k_i incurs a utility cost $C(k_i)$, which is assumed to be differentiable, increasing and convex. This cost is meant to capture resources required to screen more innovative ideas in

[2]In the appendix, I show how the process of project selection can be modelled explicitly.

financing startups, more reliable borrowers in loan markets, or unexploited arbitrage opportunities in asset markets.

After selecting k_i specialized projects, the manager can allocate k_i units of capital to these projects and the remaining $K_i - k_i$ to standard projects, generating a total payoff

$$\begin{aligned} \pi_i &= r_{i,j}k_i + R_i (K_i - k_i) \\ &= \bar{R}K + (r + \varepsilon_i) k_i + \omega (K - k_i) + u_i K, \end{aligned} \tag{1.1}$$

where $r \equiv \bar{r} - \bar{R}$. A higher k_i implies that the total payoff is more sensitive to the sector-specific shock and less to the aggregate shock. I assume that ω has cdf F_ω, with mean 0 and variance σ_ω^2, and the other random variables are Gaussian, $\varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right)$, $u_i \sim N\left(0, \sigma_u^2\right)$, $\forall i$. All random variables are assumed to be independent of each other.

I assume that $\bar{r} > \bar{R}$ so specialized projects have a higher expected payoff and are uncorrelated with the aggregate shock ω. So investors will strictly prefer specialized projects. However, selecting specialized projects requires costly, unobservable effort by the manager. This is the source of the agency problem in the model.

At time 0, after the contract is signed, the manager has access to a Walrasian market where he can trade securities contingent on the sector-specific shocks ε_j and on the aggregate shock ω. Denote by $z_{j,\hat{\varepsilon}}$ the Arrow security that pays one unit of consumption at time 1 when the realization of the idiosyncratic shock ε_j is $\hat{\varepsilon}$. Similarly, $z_{\hat{\omega}}$ denotes the security that pays one unit of consumption at time 1 when the realized aggregate state is $\hat{\omega}$. Let $\mathcal{Z}^\varepsilon$ and $\mathcal{Z}^\omega$ be the space of Arrow securities contingent on ε-risk and ω-risk, respectively, and let $\mathcal{Z} = \mathcal{Z}^\varepsilon \cup \mathcal{Z}^\omega$. Let $p : \mathcal{Z} \to \mathbb{R}_+$ be the price schedule of these securities. Denote by $d_i : \mathcal{Z} \times \mathbb{R}_+ \to \mathbb{R}$ the demand of Arrow securities $z \in \mathcal{Z}$ at price p by agent i.

To simplify notation, without loss of generality, I will assume that manager i is matched with sector i.

Payoffs. Securities are traded in a competitive market at the equilibrium price $p(\cdot)$. Managers decide the amount k_i to invest in the specialized projects and the quantities of Arrow securities to trade.

The final profits generated by manager i with demand d_i of Arrow securities are

$$\Pi_i^m = \pi_i + \int (z_{j,\hat{\varepsilon}} - p_{j,\hat{\varepsilon}})\, d_{i,j,\hat{\varepsilon}}\; d\hat{\varepsilon}\; dj + \int (z_{\hat{\omega}} - p_{\hat{\omega}})\, d_{i,\hat{\omega}}\; d\hat{\omega} \tag{1.2}$$

These profits are delivered to investors who then make a payment ξ_i to manager i. This payment depends on the what investors can observe as stated in Assumption 1. Each manager chooses an investment fraction k_i and a demand schedule d_i so as to maximize the expected utility

$$\max_{k_i, d_i} \mathbb{E}\left[u\left(\xi_i\right)\right] - C\left(k_i\right),$$

where the expectation is taken over the realizations of ξ_i.

Let $y : \mathcal{Z} \times \mathbb{R}_+ \to \mathbb{R}$ be the quantity of security z supplied by the representative investor at price $p(z)$. The profits from selling securities are given by

$$\Pi^I = \int (p_{j,\hat{\varepsilon}} - z_{j,\hat{\varepsilon}})\, y_{j,\hat{\varepsilon}}\ d\hat{\varepsilon}\ dj + \int (p_{\hat{\omega}} - z_{\hat{\omega}})\, y_{\hat{\omega}}\ d\hat{\omega}$$

The representative investor receives profits from managers and makes payments ξ_i to each manager. Therefore, in period 1 his consumption is $c(\omega) = \int (\Pi_i^m - \xi_i (\Pi_i^m, \omega))\, di + \Pi^I$ and his marginal utility of consumption in state ω is $m(\omega) \equiv v'(c(\omega))$. In equilibrium, the representative investor is fully diversified across managers. Diversification across managers also implies that it is optimal for the representative investor to write a contract with each manager separately and solve:

$$\max_{y, \xi_i} \left(\mathbb{E}\left[m(\omega)(\Pi_i^m - \xi_i)\right] + \mathbb{E}\left[m(\omega)\,\Pi^I\right] \right).$$

Information. To complete the description of the model, I need to make assumptions on the information sets of the different agents.

Assumption 1. *(a) The effort k_i, the matching F, the demand d_i, and the shock u_i are observed only by the managers and not by the investors.*

(b) The incentive contract cannot be a function of ε_j.

(c) The random variables ε_i, $\forall i$, and ω are realized at time 1 *and observed by* every agent.

The fact that effort k_i is not observable (part (a)) is the key moral hazard problem: without the right incentives, a manager will avoid paying the non-monetary cost by investing all the capital in standard projects.

The complexity of the balance sheets of banks and their trading activity is captured with two assumptions. First, part (a) implies that investors cannot condition their incentives on the trading activity of the managers. Second, part (b) restricts the space of contracts available to investors. Even if the realizations of the shocks ε_j are observable (part (c)), investors cannot condition their payments on these shocks. This prevents them from designing a contract that elicits information about the sector j to which each manager is matched. The first two assumptions together imply that the payment ξ_i can be conditioned only on the profits of the managers Π_i^m, $\forall i$, and the aggregate shock ω. These assumptions will imply that securities will have ambiguous effects on the quantity of aggregate risk and welfare in the economy. Anticipating some results, if investors could observe the trades of securities, they would always forbid trading of aggregate risk. Securities contingent on aggregate risk distort the incentives of the managers away from the desired solution and act as a constraint on the incentives that can be provided to managers.

In section 1.6, I replace part (a) with the assumption that investors can observe the quantity of securities traded by managers, but not the *type* of securities. This alternative assumption is motivated by the idea that, while investors can often observe whether financial institutions are trading securities, they may not have the expertise to understand what risks are being hedged with these complex securities.

I make the following assumptions on the utility and cost functions.

Assumption 2.

(a) *The utility function* $u(\cdot)$ *is such that* $\tilde{u}(x) \equiv (u')^{-1}(1/x)$ *is increasing and concave.*

(b) *The utility* $u(\cdot)$ *and cost* $C(\cdot)$ *functions are such that* $u(\lambda x) - C(\lambda x) = h(\lambda)(\tilde{u}(x) - C(x))$, *for some positive function* $h(\cdot)$.

Part (a) of assumption 2 is basically an assumption on the curvature of the utility function of the agent which is discussed in Jewitt (1988)[3]. This assumption is used to justify the first-order approach. Also, as it will become clear in section 1.5.1, this assumption implies that the agent will want to buy full insurance against the idiosyncratic risk.

Part (b) is a homogeneity property that serves an important purpose. This assumption and the fact that the profits (1.1) are proportional to the capital invested imply that the incentives problem for each manager will be invariant to the quantity of capital invested. In other words, under this assumption, incentives are invariant on how managers distribute capital across managers. Thus, in equilibrium investors will give the same amount of capital to each manager and fully diversify their investment. Finally, if investors are diversified across managers, I can simplify the problem by solving for the optimal contract of each manager separately.

Managers and investors meet in a Walrasian market to trade Arrow securities. The assumption of a Walrasian market deserves some comments. Most complex financial securities are traded on OTC markets where the seller and the buyer trade in a decentralized fashion. In this sense, the choice of a Walrasian environment is not very realistic and there is a growing literature that dispenses with the Walrasian assumption and focuses on decentralized markets (Duffie et al. (2005)). However, while models of decentralized trading would describe the functioning of OTC markets more realistically, they would also greatly complicate the analysis without changing the main message of the model. The focus of this paper is on the effects of complex securities on investment choices and not on the specific features of the market where these securities are exchanged. Also, the conclusions of this paper are likely to hold under different trading arrangements as long as the different types of risks are hedgeable and some trades cannot be observed.

[3] Rogerson (1985), Sinclair-Desgagne (1994), and Conlon (2009) study other conditions for the first-order approach to be valid.

Finally, the intermediation role of financial institutions is only implicit: managers borrow money from investors and run the projects themselves. There is, however, an alternative interpretation which leads to similar conclusions. Investors lend money to financial institutions which then channel this money to entrepreneurs who can select and run projects. In this more general setting there is room for two layers of moral hazard. Financial intermediaries have the expertise to monitor the entrepreneurs (Diamond (1984)) who, in turn, need incentives to make the right investment choice (that is, projects with higher return and lower correlation)[4].

Equilibrium

The equilibrium of the model is a combination of a standard Walrasian equilibrium and an optimal contracting problem between principals and agents.

Since the problem of every agent is perfectly symmetric, I restrict attention to a symmetric equilibrium where all the investors and managers make the same choice. Under assumption 2, in equilibrium investors will fully diversify their investments by lending an equal share of their endowment to each manager. Thus, investors will care only about the mean return and the aggregate risk of their portfolio and consume $c(\omega)$, which is a function of only the aggregate state.

Definition 1 (Contract). *Given a price schedule $p(\cdot)$, a contract between a principal and a manager i is a tuple (k_i, d_i, ξ_i) where k_i is the suggested level of investment in the specialized projects, d_i is the suggested demand schedule of the different securities, and $\xi_i : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is the payment made to the manager when (π_i, ω) is observed.*

An agent who behaves as specified by a contract (k_i, x_i, ξ_i) receives utility

$$\int u(\xi_i(\pi_i, \omega))\, dF_{\pi_i,\omega}(\pi_i, \omega | k_i, d_i, p(\cdot)) - C(k_i),$$

where $F_{\pi_i,\omega}(\pi_i, \omega | k_i, d_i, p(\cdot))$ is the cdf of the joint distribution of (π_i, ω) when the specified allocations are (k_i, d_i) and securities are priced according to $p(\cdot)$. A contract (k_i, d_i, ξ_i) is individually rational (IR) if it delivers utility at least equal to $\bar{u}$ to agent i, that is,

$$\int u(\xi_i(\pi_i, \omega))\, dF_{\pi_i,\omega}(\pi_i, \omega | k_i, d_i, p(\cdot)) - C(k_i) \geq \bar{u}.$$

A contract is incentive compatible (IC) if the agent doesn't want to deviate by choosing different quantities $\left(\hat{k}_i, \hat{d}_i\right)$, formally

$$(k_i, d_i) \in \arg\max_{\left(\hat{k}_i, \hat{d}_i\right)} \int u(\xi_i(\pi_i, \omega))\, dF_{\pi_i,\omega}\left(\pi_i, \omega | \hat{k}_i, \hat{d}_i, p(\cdot)\right) - C\left(\hat{k}_i\right).$$

[4]See, for example, Holmstrom and Tirole (1997) and Brunnermeier and Sannikov (2011).

Denote by $\mathcal{C}(p(\cdot))$ the set of contracts which are IR and IC when the equilibrium price schedule is $p(\cdot)$.

Definition 2 (Equilibrium). *An equilibrium is a price schedule $p(\cdot)$, contracts $(k_i, d_i, \xi_i) \in \mathcal{C}(p(\cdot))$, $\forall i$, and supply schedule y such that:*

(a) Given prices $p(\cdot)$, (k_i, d_i, ξ_i, y) is optimal for the investors;

(b) Prices $p(\cdot)$ are such that securities markets clear: $\int d_i(z, p(z))\, di = y(z, p(z))$, $\forall z$.

The definition of equilibrium essentially requires that every agent optimizes by taking the pricing function $p(\cdot)$ as given and markets clear.

1.4 No contingent securities

This section studies a simpler version of the model where markets for securities are absent. The goal of this section is twofold: it helps gain intuition before solving the full model and it represents an important benchmark for the full model's solution.

With no trades of securities, the only frictions in the economy are the fact that only managers observe the fraction of wealth invested in specialized projects. I show that the agency problem reduces the level of investment in the specialized projects that is implemented in equilibrium. In turn, this increases aggregate volatility in the economy and lowers welfare. However, this is not a reason for policy intervention as the equilibrium without trading of securities is constrained efficient.

When securities markets are absent the model and equilibrium definition are as in section 1.3, except that the demand and supply schedules d_i, y, and the price $p(\cdot)$ are absent.

Let k_i be the level of specialized investment that the principal wants to implement in equilibrium. It is convenient to rewrite the problem by considering the following transformation of π_i

$$x_i = \frac{\pi_i - \bar{R} - r\ k_i - \omega(1 - k_i)}{\sigma_x},$$

where $\sigma_x = \sqrt{k_i^2 \sigma_\varepsilon^2 + \sigma_u^2}$ is the idiosyncratic volatility of π_i if the manager chooses k_i. Suppose now that the investor recommends an investment level k_i to manager i, but the latter deviates to a fraction $\hat{k}_i \neq k_i$. Then, the distribution of x_i will in general depend on both k_i and $\hat{k}_i$. Also, when $\hat{k}_i = k_i$, x_i is the linear projection of π_i on the space orthogonal to ω and, hence, x_i is uncorrelated with ω. Moreover, by the Gaussian assumption, x_i turns out to be the best predictor of the idiosyncratic part of π_i.

The distribution of x_i conditional on ω when the recommended fraction is k_i but the manager deviates to $\hat{k}_i$ is also Gaussian with with mean and variance given by

$$\mu_{x|\omega} = \frac{r - \omega}{\sigma_x}\left(\hat{k}_i - k_i\right), \quad \sigma_{x|\omega}^2 = \frac{1}{\sigma_x^2}\left(\hat{k}_i \sigma_\varepsilon^2 + \sigma_u^2\right). \tag{1.3}$$

In equilibrium where $\hat{k}_i = k_i$, we have $\mu_{x|\omega} = 0$ and $\sigma^2_{x|\omega} = 1$. This is the reason why this transformation makes it easier to solve the problem.
If we let $F_{x|\omega}\left(x_i|\omega; k_i, \hat{k}_i\right)$ be the cdf of this conditional distribution, then in equilibrium $F_{x|\omega}\left(x_i|\omega; k_i, \hat{k}_i\right)$ doesn't depend on ω nor on k_i and $F_{x|\omega}\left(x_i|\omega; k_i, \hat{k}_i\right) = \Phi(x)$, where $\Phi(x)$ is the cdf of a standard Gaussian distribution.

With this transformation, the contracting problem with no securities solves

$$\max_{\xi,k} \int m(\omega)\left(x + \bar{R} + r\ k + (1-k)\,\omega - \xi(x,\omega)\right) d\Phi(x)\, dF_\omega(\omega) \tag{P(NS)}$$

subject to:

$$k \in \arg\max_{\hat{k}} \int u\left(\xi(x,\omega)\right) dF_{x|\omega}\left(x|\omega; k, \hat{k}\right) dF_\omega(\omega) - C\left(\hat{k}\right), \tag{IC}$$

$$\int u\left(\xi(x,\omega)\right) d\Phi(x)\, dF_\omega(\omega) - C(k) \geq \bar{u}. \tag{IR}$$

The investor maximizes the final payoff of the investment weighted by his marginal utility of consumption subject to two constraints. The first constraint requires that the compensation scheme and the recommended efforts are such that the manager finds it optimal to comply with the recommendation. The second constraint is the usual IR constraint.

The common strategy in the moral hazard literature is to relax problem P(NS) by replacing the IC constraint with its first-order condition. Formally, I can replace IC by

$$\frac{\partial}{\partial \hat{k}} \int u\left(\xi(x,\omega)\right) dF_{x|\omega}\left(x|\omega; k, \hat{k}\right) dF_\omega(\omega)\bigg|_{\hat{k}=k} - C'(k) = 0. \tag{IC'}$$

The big advantage is that we can now use Lagrangian methods and solve P(NS) by taking first-order conditions. Let λ and μ be the Lagrange multipliers on the IR and IC' constraints, respectively. The following proposition characterizes the optimal contract.

Proposition 1. *Let k be the investment fraction that the principal wants to implement. The optimal contract for the model with no contingent securities solves*

$$\frac{m(\omega)}{u'\left(\xi(x,\omega)\right)} = \mu + \lambda\left[\frac{1}{\sigma_x}(r-\omega)\,x + \frac{k\ \sigma^2_\varepsilon}{\sigma^2_x}\left(x^2 - 1\right)\right]. \tag{1.4}$$

To gain some intuition on the optimal contract (1.4) we can compare it to the case with no agency problem where investors are allowed to observe also k_i (I refer to this case as the "first-best"). If investors can observe k_i, they will severely punish the manager who doesn't comply with the recommendation. Assuming that the punishment can be made severe enough that we can drop the IC constraint from the problem we have that the first-best contract will be given by (1.4) with

$\lambda = 0$[5]. Thus, the first-best optimal contract simply allocates aggregate risk between the two risk-averse agents and the optimal payment schedule does not depend on the realization of π (Stoughton (1993)). However, since the manager incurs in the cost $C(k)$, even in the first-best the optimal choice of k may be different from 1 and there may be some aggregate risk in the economy.

The form and the interpretation of (1.4) is made easy by the assumption of Gaussian random variables. The contract has two main components. First, the usual risk-sharing component given by the left-hand side of (1.4). This term determines how aggregate risk is shared between the investor and the manager depending on the curvature of their utility functions. This term was the only piece in the first-best contract which completely insulated the agent from the idiosyncratic risk. However, to provide the manager with the right incentives, the payment schedule has to be a function of the new variable x. Incentives are provided through the right-hand side of (1.4).

The optimal contract has the same structure as that obtained by Holmstrom (1979), who shows that the best way to incentivize the agent is to make his payment conditional on the likelihood ratio of his action. The Gaussian assumption for ε_i and u_i delivers the simple expression for the likelihood ratio given by the the term that multiplies λ in (1.4). From (1.3) we know that, if the agent invests in the specialized projects an amount $\hat{k}_i$ that is slightly lower than the suggested k_i, this will have three effects on the distribution of x_i. First, the mean of x_i will be lower. Thus, when observing a lower realization of x_i the principal should infer a deviation by the agent and punish him accordingly. This explains the term $r\,x_i$ in the right-hand side of (1.4). Secondly, when a lower $\hat{k}_i$ is selected, the distribution of x_i will be correlated with ω. Thus, a comovement between x_i and ω is a signal of a possible deviation and, thus, the optimal contract punishes the agent (this explains the term $\omega\, x$ in the contract). Finally, a lower choice of $\hat{k}_i$ also reduces the volatility of x_i and so the contract rewards the agent for realizations of x_i which are far from its mean. This is the reason why the convex term x^2 enters the contract. From (1.3) we know that in equilibrium the variance of x is 1, hence the optimal contract rewards the agent for realizations of x^2 relative to this value. In equilibrium the term that multiplies λ in (1.4) has mean 0 (this is a general property of likelihood ratios).

The optimal contract uses the aggregate state ω to provide the agent with incentives. In equilibrium, the investor conditions the payment of each manager to the average performance of the other managers in the economy. This type of benchmarking, however, is different from the result stressed in the moral hazard literature with common noise following Holmstrom (1979). The latter also considers a principal-agent problem with multiple agents and correlated risk. He assumes that $\pi_i = (r + \varepsilon_i)\, k_i + \omega + u_i$, that is, the choice of the agent doesn't affect the amount of aggregate risk in the project. With this alternative payoff structure the model of this paper would reproduce the classical result that, when the aggregate state is known, the optimal contract should not be

[5]Note that the first-best Lagrange multiplier μ_{FB} will differ from μ in (1.4).

conditioned upon it. Intuitively, more risk that is not related to the agent's effort only makes it harder to incentivize a risk-averse agent[6].

The agency problem makes it more expensive for the principal to implement a certain value of k. Thus, it is natural to expect a lower value of k to be implemented in equilibrium.

Proposition 2. *When the choice of k is not observable, equilibrium k is lower (and aggregate volatility is higher).*

The agency problem, therefore, cause the volatility of the economy to increase. However, the higher volatility is not a symptom of inefficiency. A social planner who has access to the same information as the investors (that is, the planner also faces the same agency problem) cannot improve on the equilibrium.

Proposition 3. *The equilibrium outcome of the economy is efficient.*

The agency problem causes the economy to be more volatile and yet there is no room for policy intervention. This conclusion will change radically when agents will be allowed to trade securities.

1.5 Trades of Securities

In this section, I consider the full model where managers can trade securities contingent on the different risks in the economy. Some assumptions on the distributions of the shocks guarantee that managers will find it optimal to trade contingent securities and hedge their risks. One of the main conclusions of this model is that, under certain conditions, trading of securities has dramatically different implications for aggregate volatility and welfare depending on whether idiosyncratic or aggregate risk is traded. More specifically, I show that investors are better off when managers pool and eliminate their exposure to idiosyncratic risks.

These conclusions change substantially when aggregate risk is considered. By definition, aggregate risk cannot be pooled and eliminated, but some agents have to ultimately bear it. Also, in a symmetric equilibrium, managers receive the same contract and, thus, bear the same amount of aggregate risk. Thus, the only gains from trading securities on aggregate risk are possible only if investors participate in the market. The fact that investors are willing to take some aggregate risk might seem unreasonable. After all, investors are those who design the contracts that exposes

[6]We can see this in my model by observing that with this new definition of π_i (1.4) becomes:

$$\frac{m(\omega)}{u'(\xi(x,\omega))} = \mu + \lambda\left[\frac{1}{\sigma_x} r\, x + \frac{k\,\sigma_\varepsilon^2}{\sigma_x^2}\left(x^2 - 1\right)\right],$$

and the aggregate state disappears from the incentives component of the contract (ω appears only through the risk-sharing component). As expected, the principal doesn't uses aggregate risk to incentivize the agent. If in addition the principal was risk-neutral, then he would completely insulate the agent from aggregate risk.

managers to some aggregate risk to incentivize them. Indeed, if investors could affect the amount of ω-securities that are traded by managers, they would forbid these trades. However, by assumption 1, investors do not observe and, hence, cannot contract upon the trades investors make. Even if investors cannot observe the trades of securities, they will design a new contract that takes these trades into account. In particular, they will change the optimal contract so that it will not be optimal for managers to trade ω-securities in equilibrium. Thus, while ω-securities will not be traded on equilibrium, the possibility of trading these securities will act as a constraint on the optimal contracting problem. This stands in contrast to the case with ε-securities, which are traded on equilibrium.

The amount of insurance bought by managers depends on its equilibrium price. In the case of ε-securities, the possibility of eliminating risks by pooling them together allows for insurance to trade at an actuarially fair price[7]. If actuarially fair insurance is at least possible for securities contingent on idiosyncratic risk, this is no longer true when aggregate risk is traded. Intuitively, this risk has to shared between investors and managers and the price of insurance will depend, among other things, on their marginal utility of consumption.

Since they have potentially different effects on the principal-agent problem, it is helpful to first analyze ε-securities and ω-securities separately.

1.5.1 Securities on idiosyncratic risk

I first focus on securities contingent on idiosyncratic risk and forbid trades of securities contingent on the aggregate state. By assumption 1, investors cannot observe the trades made by managers and, thus, they can't condition the payment schedule on this information. I start with the characterization of the equilibrium price $p(\cdot)$ and then solve for the optimal contract. Markets are assumed to be competitive and all the agents take the price schedule $p(\cdot)$ as given.

When ω-securities are not allowed, the final payoff (1.2) of manager i who invests k_i in the specialized projects and buys a quantity $d_{i,j,\hat{\varepsilon}}$ of security $z_{j,\hat{\varepsilon}}$ at price $p_{j,\hat{\varepsilon}}$ is

$$\Pi_i^m = \pi_i + \int \left(z_{j,\hat{\varepsilon}} - p_{j,\hat{\varepsilon}}\right) \, d_{i,j,\hat{\varepsilon}} \, d\hat{\varepsilon} \, dj,$$

Let $F_{\Pi_i,\omega}\left(\Pi_i, \omega | k_i, d_i, p(\cdot)\right)$ be the distribution of the pair (Π_i, ω) for given choice of k_i, demand schedule d_i, and price schedule $p(\cdot)$.

The agent now chooses both the investment k_i and the demand schedule d_i for given price

[7] Of course, in the presence of a cost to trade securities (as in sections 1.6 and 1.7), market power, or other frictions, the price of insurance would deviate from the the actuarially fair price.

schedule $p(\cdot)$. The IC constraint for the contracting problem becomes:

$$(k_i, d_i) \in \arg\max_{\hat{k},\hat{d}_i} \int u(\xi_i(\Pi_i, \omega))\, dF_{\Pi_i,\omega}\left(\Pi_i, \omega | \hat{k}_i, \hat{d}_i, p(\cdot)\right) - C\left(\hat{k}_i\right) \tag{1.5}$$

As for the analysis of section (1.4), it is convenient to consider the linear projection of Π_i^m onto the space orthogonal to ω. Formally, define x_i as follows:

$$x_i = \frac{\Pi_i^m - \bar{R} - r\, k_i - \omega(1-k_i) + \int p_{j,\hat{\varepsilon}}\, d_{i,j,\hat{\varepsilon}}\, d\hat{\varepsilon}\, dj}{\sigma_{i,x}}, \tag{1.6}$$

where $\sigma_{i,x}^2 \equiv Var(x_i)$ is the equilibrium variance of x_i (that is, when $\hat{k}_i = k_i$).

Let Φ_ε be the cdf of a Gaussian distribution with mean 0 and variances σ_ε^2. As lemma 1 shows, the equilibrium price is such that the idiosyncratic risk is traded at an actuarially fair price.

Lemma 1. *The equilibrium price of an Arrow security $z_{i,\hat{\varepsilon}}$ is $p_{i,\hat{\varepsilon}} = \phi_\varepsilon(\hat{\varepsilon})$.*

The problem is now whether the principal wants the agent to buy full insurance at the price of lemma 1. The answer is complicated by the fact that the idiosyncratic shock ε_i multiplies k_i in the final payoff of the manager. Thus, a more volatile ε_i can potentially convey some information about the actual choice of k_i. Thus, it may be optimal for the principal if the manager traded securities so as to increase the variance of ε_i. This reasoning applies only to the securities contingent on the shock of the sector that is matched to the manager. If a manager traded securities conditional on the shocks of other sectors, this would only add noise to his profits and worsen the agency problem.

The main complication with the fact that a more volatile ε_i contains information about k_i is that we are allowing any ε-security to be traded. By trading in the securities market, the manager can buy ε-securities that change the distribution of ε. The distribution of (1.6) will not be necessarily Gaussian, since the agent can potentially demand any quantity $d_{i,j,\hat{\varepsilon}}$ of any security $z_{j,\hat{\varepsilon}}$. The principal has to incentivize manager i to choose a demand schedule d_i and, thus, a whole distribution $F_{\Pi_i,\omega}(\Pi_i, \omega | k_i, d_i, p(\cdot))$. Thus, the quantity of securities demanded by the agent depends on the optimal contract which, in turn, has to be chosen by the principal so that the agent demands the right amount of securities.

When full insurance is optimal for the principal the problem becomes simpler. To see this in a more formal way, note that an agent wants to buy full insurance whenever his payment schedule ξ makes his problem concave in x. Conjecture now that the agent buys full insurance and solve for the optimal ξ. If this payment schedule makes the problem of the agent concave, then the conjecture is verified and we have found the solution to original the problem.

The problem is then to find conditions under which the principal wants the agent to buy full insurance. It is easy to see that full insurance would be optimal in the absence of the error term u_i

in the payoff of the manager. If u_i was absent, full insurance would make the choice of k_i perfectly observable and the agency problem would disappear. This would lead to the first-best outcome which, by definition, is the best outcome for the principal. On the other side, suppose that the volatility of ω is close to 0 and so is the mean r. In this case, it is harder for the principal to identify the value of k_i chosen by the agent. A more volatile ε_i makes the distribution of Π_i^m more sensitive to k_i and helps the principal.

In the appendix, I derive a condition for full insurance to be optimal. This condition is related to the volatility of the likelihood ratio of the distribution $F_{x_i|\omega}$. Intuitively, the likelihood ratio can be seen as a measure of how informative are the signals about the choice of k_i. Signals are more informative when the likelihood ratio is more volatile and, therefore, a more volatile likelihood ratio leads to a better outcome for the principal. The condition in the appendix requires that the likelihood ratio is most informative when the variance of σ_ε^2 is 0, that is, when the agent is fully insured. This condition is more likely to be satisfied for higher values of r, for lower values of σ_u^2, and for higher values of σ_ω^2. This confirms the intuition in the discussion above.

The problem would be simpler if the agent was allowed to trade only *linear* securities, that is, securities with a payoff $q\ \varepsilon_i$, for some scalar q. These securities, in fact, preserve the normality of the $F_{x_i|\omega}$ and I can derive intuitive sufficient conditions on the parameters of the model for which full insurance is optimal.

Lemma 2. *Assume that only linear ε-securities can be traded. If $0 \leq \sigma_u^2 \leq (r-\omega)^2, \forall\omega$, then full insurance is optimal.*

The condition is easy to interpret. Take for example $r = 0$. Then the condition says that full insurance is optimal whenever the realizations of the random variable ω are "big" enough[8]. This sufficient condition may seem restrictive because it has to hold for any problem (of course, under the assumption of linear securities). For each specific problem, however, it is possible to weaken this assumption (for example, to substitute it with some appropriate average of ω).

Let's conjecture that it is optimal for the agent to buy full insurance. Formally, this means that an agent who invest k_i in the specialized project will demand $-\hat{\varepsilon}\ k_i$ units of the Arrow securities $z_{i,\hat{\varepsilon}}$, $\forall\hat{\varepsilon}$, and zero units of all the other securities. Also, from lemma 1, we know that the cost of this insurance is

$$\int p_{i,\hat{\varepsilon}}\ d_{i,j,\hat{\varepsilon}}\ d\hat{\varepsilon} = -k_i \int \hat{\varepsilon}\ d\Phi_\varepsilon(\hat{\varepsilon}) = 0$$

Combining these two results implies that the profits of the agent are given by

$$\Pi_i^m = \pi_i - k_i\varepsilon_i$$

[8]Remember that ω is not restricted to be continuous, but it can be a discrete random variable with mean 0. For example, $\omega_H > 0 > \omega_L = -\omega_H$.

Similarly, (1.6) becomes

$$x_i = \frac{\Pi_i^m - \bar{R} - r\ k_i - \omega\,(1 - k_i)}{\sigma_u}.$$

As usual, in equilibrium where the agent trades $-k_i\varepsilon_i$ the random variables x_i and ω are uncorrelated.

With a slight abuse of notation, let $F_{x_i|\omega}\left(x_i|\omega; k_i, \hat{k}_i, -k_i\varepsilon_i, p(\cdot)\right)$ be the cdf of the Gaussian random variable x_i conditional on ω, when the agent chooses $\hat{k}_i$ and trades $-k_i\varepsilon_i$. Note that when the agent buys full insurance, the distribution $F_{x_i|\omega}\left(x_i|\omega; k_i, \hat{k}_i, -k_i\varepsilon_i, p(\cdot)\right)$ will not depend on $p(\cdot)$. However, out of the equilibrium, if the agent deviates to a different portfolio allocation, then this distribution will depend on the price schedule $p(\cdot)$.

The moments of $F_{x_i|\omega}\left(x_i|\omega; k_i, \hat{k}_i, -k_i\varepsilon_i, p(\cdot)\right)$ are given by

$$\mu_{x|\omega} = \frac{r-\omega}{\sigma_u}\left(\hat{k}_i - k_i\right), \quad \sigma^2_{x|\omega} = \left(\hat{k}_i - k_i\right)^2 \frac{\sigma_\varepsilon^2}{\sigma_u^2} + 1 \tag{1.7}$$

In equilibrium, $\hat{k}_i = k_i$ and the agent trades $-k_i\varepsilon_i$, so the moments are $\mu_{x|\omega} = 0$ and $\sigma^2_{x|\omega} = 1$. Thus, once again we have that $F_{x|\omega}\left(x|\omega; k_i, k_i, -k_i\varepsilon_i, p(\cdot)\right) = \Phi(x)$.

We are now ready to solve the optimal contracting problem where the agent buys full insurance and both the principal and the agent take the price of ε-securities as given. As shown in (1.5), the agent now faces two choices. First, he has to decide what fraction k_i to invest in the specialized projects. Second, he has to decide the quantity of Arrow securities to trade.

Formally, the optimal contract solves (omitting subscripts i for convenience):

$$\max_{\xi,k,d^\varepsilon} \int m(\omega)\left(x + \bar{R} + r\ k + (1-k)\,\omega - \xi(x,\omega)\right) d\Phi(x)\, dF_\omega(\omega) \tag{P(ε)}$$

subject to:

$$(k,d) \in \arg\max_{\hat{k},\hat{d}} \int u\left(\xi(x,\omega)\right) F_{x|\omega}\left(x|\omega; k, \hat{k}, \hat{d}, p(\cdot)\right) - C\left(\hat{k}\right), \tag{IC}$$

$$\int u\left(\xi(x,\omega)\right) d\Phi(x)\, dF_\omega(\omega) - C(k) \geq \bar{u}. \tag{IR}$$

This problem is similar to P(NS) in the case with no securities, except that now the IC constraint takes into account the two choices of the agent. Under the assumptions that make full insurance optimal, we can considerably simplify this problem. To see this, relax problem P(ε) by dropping the constraint IC on the choice of d_i. If the contract that solves the relaxed problem is such that the agent wants to buy full insurance then this contract must also solve the original problem with the full IC constraints. Formally, we look for the values of ξ and k that solve

$$\max_{\xi,k} \int m(\omega)\left(x + \bar{R} + r\ k + (1-k)\,\omega - \xi(x,\omega)\right) d\Phi(x)\, dF_\omega(\omega)$$

subject to:

$$k \in \arg\max_{\hat{k}} \int u\left(\xi\left(x,\omega\right)\right) F_{x|\omega}\left(x|\omega; k, \hat{k}, -k\ \varepsilon, p\left(\cdot\right)\right) - C\left(\hat{k}\right),$$

$$\int u\left(\xi\left(x,\omega\right)\right) d\Phi\left(x\right) dF_{\omega}\left(\omega\right) - C\left(k\right) \geq \bar{u}.$$

Once again, I conjecture that the FOA is valid for this problem, relax the IC constraint by replacing it with its first-order condition, and then verify that this conjecture is valid at the optimal contract. The FOA allows us to solve for the optimal contract using Lagrangian methods as shown in the next proposition.

Proposition 4. *Let λ and μ be the Lagrange multipliers on the IC and IR constraints, respectively, and suppose full insurance is optimal. The optimal payment schedule $\xi\left(x,\omega\right)$ satisfies:*

$$\frac{m\left(\omega\right)}{u'\left(\xi\left(x,\omega\right)\right)} = \mu + \lambda \frac{1}{\sigma_u}\left(r - \omega\right)\ x. \tag{1.8}$$

Proposition 4 immediately implies that the approach of relaxing the IC constraint and then verify that the agent wants to buy full insurance is valid. This follows from assumption 2 which guarantees that under the optimal contract (1.8) the agent's problem is concave. In turn, concavity implies that the agent wants to buy full insurance at the actuarially fair price of lemma 1. Concavity of the agent's problem also implies that the FOA is valid for this problem (Jewitt (1988)).

The shape of the new optimal contract is similar to (1.4) and, not surprisingly, the main difference is the absence of the convex term x^2. In equilibrium, the agent buys full insurance against his idiosyncratic risk and, thus, the principal does not reward him if his profits are volatile.

To gain more intuition about proposition 4 and about how ε-securities affect the equilibrium, I allow the principal to use ε_i when providing incentives to the agent. Formally, I relax part (a) and part (b) of assumption 1 and allow the principal to write a contract on the shock of the specific sector to which the manager is matched. Thus, the payment ξ_i can be conditioned on both shocks ω and ε_i. Similarly to the case with ω-securities analyzed in the following section, if the principal can observe the realization of ε_i then it is easy to see that there are no gains from allowing trades of ε-securities. Instead, the presence of ε-securities can only hurt investors to the extent that they cannot limit these trades. However, the scope of this exercise is to compare the equilibrium of the model where the principal provides incentives through the financial markets to case where the principal can contract on ε_i directly. Therefore, I also forbid trades of securities.

Formally, the contracting problem is similar to P(NS) of section 1.4 with the difference that now ξ can also be a function of ε. I can then define x as follows:

$$x_i = \frac{\pi_i - \bar{R} - r\ k_i - \omega\left(1 - k_i\right) - \varepsilon_i k_i}{\sigma_u},$$

so that, in equilibrium where $\hat{k}_i = k_i$, I have that $\pi_i = u_i/\sigma_u$. The next lemma describes the optimal contract under these new assumptions.

Lemma 3. *Let λ and μ be the Lagrange multipliers on the IC and IR constraints, respectively. The optimal payment schedule $\xi(x, \omega, \varepsilon)$ satisfies:*

$$\frac{m(\omega)}{u'(\xi(x, \omega, \varepsilon))} = \mu + \lambda \frac{1}{\sigma_u}(r - \omega + \varepsilon)\ x. \tag{1.9}$$

The contract (1.9) treats the two shocks ω and ε_i symmetrically. The agent is punished if x_i is correlated with the aggregate state ω and is *rewarded* if x_i is correlated with the idiosyncratic shock ε_i. In fact, a correlation between x_i and ε_i is a sign that the agent selected $\hat{k}_i > k_i$. Lemma 3 shows that, when the principal can contract on ε_i, he will choose a contract that differs from (1.8). The principal does not fully insure the manager against the idiosyncratic risk, but exposes him to some ε-risk. Of course, under the assumptions of lemma 3 the principal is better off relative to the case of proposition 4.

Lemma 3 is interesting also for another reason. Suppose that the principal has access to some information about ε_i. For example, suppose that the principal receives a partially informative signal about the identity i of the specialized investment and he can use this signal in the contract. How will the principal use this information? Lemma 3 suggests that the principal will expose the agent to some ε-risk by conditioning the optimal contract to this signal and the optimal payment will resemble (1.9).

I can now state the main results of this section. The key question is what happens to the aggregate volatility of the economy and to welfare when securities contingent on ε-risk are traded in the market. Since managers face a binding IR constraint, welfare here is simply the utility of the representative investor. As the next proposition shows, under the conditions that make full insurance of the ε-risk optimal for the principal, these securities reduce aggregate volatility and increase welfare.

Proposition 5. *Securities contingent on ε-risk increase equilibrium k (and, thus, lower aggregate volatility) and increase welfare in the economy.*

As discussed above, securities on ε-risk make it easier for a principal to identify whether the agent has deviated or not. This lowers the cost of implementing higher values of k and, thus, lowers aggregate volatility. Finally, a principal who can implement higher values of k more cheaply can also achieve higher levels of welfare.

1.5.2 Securities on aggregate risk

In this section, I only allow trades of securities contingent on the aggregate state ω. This case is very different from the previous section where only the idiosyncratic risk was hedgeable. In the symmetric equilibrium considered in this paper, all the managers are perfectly symmetric and, hence, share the same quantity of aggregate risk. Thus, the only way to hedge the aggregate risk is to transfer it to investors.

From a mathematical point of view, the main difference between these two types of risks is that investors can always condition the payment schedule on aggregate risk if they find it optimal to do so. Thus, the fact that managers can trade away some of their aggregate risk should only act as a constraint on the contracting problem. I show that the possibility for managers to trade away some of the aggregate risk through ω-securities weakens the incentives that investors can provide in equilibrium. Therefore, contrary to the case where only idiosyncratic risk can be hedged, the fact that managers can transfer some risk to investors, makes the agency problem worse. A direct implication is that the existence of ω-securities reduces welfare. Again, the intuition for this result is very simple: if transferring some aggregate risk was optimal for the principal, the optimal contract would already take this into account. The key friction, therefore, is that trades are not observable. Intuitively, if investors could contract upon the quantities of ω-securities purchased by managers, they would provide incentives for the latter to stay out of this market.

Securities contingent on aggregate risk differ from those contingent on idiosyncratic risk also because the former type of risk cannot be eliminated and some agent in the economy has to ultimately bear it. In turn, this implies that the price to hedge aggregate risk cannot be actuarially fair as it was for ε-securities. In equilibrium, aggregate risk will be transferred back to the principals who are risk-averse and, thus, demand a compensation to take this risk.

The fact that the principal can always condition the payment to the aggregate state and replicate any portfolio of ω-securities chosen by the agent implies that I can focus on the case where there is no trading of ω-securities. To see this, suppose that manager i demands a quantity $d_{i,\hat{\omega}}$ of security $z_{\hat{\omega}}$ at price $p_{\hat{\omega}}$, generates profits equal to $\Pi_i^m = \pi_i + \int (z_{\hat{\omega}} - p_{\hat{\omega}}) \, d_{i,\hat{\omega}} \; d\hat{\omega}$ and obtains a payment $\xi_i (\Pi_i^m, \omega)$. The principal can always define a new payment $\tilde{\xi}_i (\pi_i, \omega) = \xi_i (\Pi_i^m, \omega)$, $\forall \pi_i$, ω, Π_i^m, such that the agent finds it optimal not to trade ω-securities. Clearly, this is suboptimal for the principal who had chosen ξ_i over $\tilde{\xi}_i$ in the first place.

Proposition 6. *The optimal contract is such that on the equilibrium path the manager does not trade ω-securities, that is, $d_{i,\hat{\omega}} = 0$, $\forall \hat{\omega}$, i.*

Thus, there are no trades in the securities market when only the ω-risk can be traded. However, even in the absence of trades, prices are still determined by the assumption of competition. The representative investor is risk-averse with stochastic discount factor $m(\omega)$. He is then willing to trade ω-securities (in fact, an infinite amount of them) whenever the price of these securities is above their marginal utility of consumption. Therefore, in equilibrium the price of these securities has to be such that the representative investor is indifferent on the quantity of securities to trade.

Lemma 4. *The equilibrium price of an Arrow security* $z_{\hat{\omega}}$ *is* $p_{\hat{\omega}} = m(\hat{\omega}) f_{\omega}(\hat{\omega}) / \mathbb{E}[m(\omega)]$.

The price of insurance against state $\hat{\omega}$, $m(\hat{\omega}) f_{\omega}(\hat{\omega}) / \mathbb{E}[m(\omega)]$, is a combination of the probability density that $\hat{\omega}$ is realized (this is the same as in the equilibrium with ε-securities) and the principal's marginal utility of consumption. The more valuable is consumption for the principal in the state of the world $\hat{\omega}$, that is, the higher is $m(\hat{\omega}) / \mathbb{E}[m(\omega)]$, the higher will be the price of a security that pays in that state.

The contracting problem with ω-securities is different from $\mathrm{P}(\varepsilon)$ in an important way. Contrary to the case with ε-securities, here we cannot conjecture that the agent doesn't want to trade the ω-risk and then verify this conjecture. In fact, if we were to relax the problem by assuming no trades of ω-securities and derive the optimal contract, this contract would not satisfy the initial conjecture: an agent receiving the payment schedule that solves the relaxed problem has an incentive to deviate and trade ω-securities. This implies that we have to explicitly incorporate the portfolio choice of the agent into the optimal contracting problem.

In the case with only ω-securities, the profits of a manager are

$$\Pi_i^m = \pi_i + \int (z_{\hat{\omega}} - p_{\hat{\omega}}) \, d_{i,\hat{\omega}} d\omega,$$

and in equilibrium where no securities are traded we have $\Pi_i^m = \pi_i$. Similarly, (1.6) becomes

$$x_i = \frac{\Pi_i^m - \bar{R} - r\, k_i - \omega (1 - k_i)}{\sigma_x}.$$

Let $F_{x_i|\omega}\left(x|\omega; k_i, \hat{k}_i, \hat{d}_i, p(\cdot)\right)$ be the conditional distribution of x_i when the agent chooses $\hat{k}_i$ and instead of k_i and demands $\hat{d}_i$. In equilibrium, the principal wants the agent to choose $\hat{d}_{i,\hat{\omega}} = 0$, $\forall \hat{\omega}$, and of course $\hat{k}_i = k_i$. Again, if the agent follows the optimal contract, the equilibrium distribution becomes $F_{x_i|\omega}\left(x|\omega; k_i, \hat{k}_i, \hat{d}_i, p(\cdot)\right) = \Phi(x)$.

Let $\bar{U}_{\omega}(k_i, p(\cdot))$ be the value of the best deviation available to the manager:

$$\bar{U}_{\omega}(k_i, p(\cdot)) = \max_{\hat{k},\, \hat{d}} \int u(\xi(x, \omega)) \, dF_{x_i|\omega}\left(x|\omega; k_i, \hat{k}, \hat{d}, p(\cdot)\right) dF_{\omega}(\omega) - C\left(\hat{k}\right)$$

So $\bar{U}_{\omega}(k_i, p(\cdot))$ is the value for a manager of deviating to a different $\hat{k}$ and a different demand schedule $\hat{d}$. Thus, to prevent the manager from deviating, the optimal contract has to be such that

$$\int u(\xi(x, \omega)) \, d\Phi(x) \, dF_{\omega}(\omega) \geq \bar{U}_{\omega}(k_i, p(\cdot)). \tag{1.10}$$

Assume that the FOA is valid, the optimal contracting problem P(ω) is given by (omitting subscripts i for convenience):

$$\max_{\xi,k,d} \int m(\omega)\left(x+\bar{R}+r\,k+(1-k)\,\omega-\xi(x,\omega)\right)d\Phi(x)\,dF_{\omega}(\omega) \tag{P(ω)}$$

subject to:

$$\frac{\partial}{\partial \hat{k}} \int u(\xi(x,\omega))\,dF_{x|\omega}\left(x|\omega;k,\hat{k},d=0,p(\cdot)\right)dF_{\omega}(\omega)\Big|_{\hat{k}=k} - C'(k) = 0, \tag{IC$_k$}$$

$$\frac{\partial}{\partial \hat{d}} \int u(\xi(x,\omega))\,dF_{x|\omega}\left(x|\omega;k,k,\hat{d},p(\cdot)\right)dF_{\omega}(\omega)\Big|_{\hat{d}=0} = 0, \tag{IC$_\omega$}$$

$$\int u(\xi(x,\omega))\,d\Phi(x)\,dF_{\omega}(\omega) - C(k) \geq \bar{u}. \tag{IR}$$

The next lemma characterizes the optimal contract with ω-securities.

Lemma 5. *Let λ, ν_ω, and μ be the Lagrange multipliers on the IC$_k$, IC$_\omega$, and IR constraints, respectively. The optimal payment $\xi(x,\omega)$ satisfies:*

$$\frac{m(\omega)}{u'(\xi(x,\omega))} = \mu + \lambda\frac{1}{\sigma_x}(r-h(\omega))\;x + \lambda\frac{k\,\sigma_\varepsilon^2}{\sigma_x^2}\left(x^2-1\right) \tag{1.11}$$

where $h(\omega)=\omega-\nu_\omega\left(1-\mathbb{E}\left[m(\omega)\right]\right)$ and $\nu_\omega \geq 0$.

Problem P(ω) shows that the presence of these securities acts as an extra constraint on the contracting problem. Intuitively, this should lead to lower welfare. Also, the presence of ω-securities makes it more costly for the principal to implement higher levels of k and, thus, aggregate volatility in the economy increases. The proof of these results is not immediate since the marginal utility of the principals $m(\omega)$ is endogenous. The next proposition confirms our original intuition and represents the main result of this section.

Proposition 7. *Securities contingent on ω-risk decrease equilibrium k (and, thus, increase aggregate volatility) and reduce welfare in the economy.*

1.6 Full model

In this section I allow both types of securities to be traded. The previous analysis showed that the two types of securities tend to have opposite effects on aggregate volatility and welfare. It is natural to expect that when we introduce both types of securities the overall effect will be ambiguous. More specifically, we can expect welfare to increase when it is possible to hedge the ε-risk and the opposite result to hold for the ω-risk.

To derive further results, I generalize the model in two ways. First, I assume that the economy is populated by a firm (which I refer to as "issuer") which creates securities. There are N issuers in the economy, denoted by ℓ, which are owned by investors. The securities created can then be traded in the Walrasian market by paying a fixed cost per trade. In general, the portfolio problem with ε-securities can become very hard to solve with most assumptions on the costs of trading securities. An easy departure from the basic model of the previous sections is to assume that, every time an issuer sells a security to a manager, the former has to pay a fixed cost[9]. This assumption has the advantage to allow for some flexibility in the cost of securities without making the model intractable. In equilibrium, prices of securities will reflect the presence of these costs. The key feature of having a fixed cost per trade is that equilibrium prices will resemble a two-part tariff, that is, the price of a security will be given by a fee (which is independent of the specific security and is high enough to cover the fixed cost) plus a term which is the same as those in lemmas 1 and 4[10].

Secondly, I assume that transactions in the financial market are observable, that is, I relax part (a) of assumption 1. As I show later, the presence of the fixed cost implies that each manager will trade at most once and with only one issuer. Thus, a transaction has to be interpreted as any trade between a manager and an issuer, independently of how many securities are exchanged.

The fact that now managers have to pay a fixed fee per trade to buy securities implies that it is optimal for them to trade with only one issuer (and, thus, pay the fee only once). For example, a manager who in the previous sections was buying two Arrow securities from two issuers (or even the same issuer), now will prefer to combine the two Arrow securities and make only one trade. Thus, I let agents trade *insurance contracts* which are general functions of the underlying Arrow securites. A manager will find it optimal to buy this insurance contracts instead of the Arrow securities to save on the trading costs.

Every insurance contract can be contingent on idiosyncratic risks or aggregate risk. Let $J^{\varepsilon} = \left\{s : \mathbb{R}^{[0,1]} \to \mathbb{R} \text{ such that } \int\int s\left(\{\varepsilon_i\}\right) d\Phi_{\varepsilon}\left(\varepsilon_i\right) di = 0\right\}$, where $\mathbb{R}^{[0,1]}$ is the set of functions from the unit interval to $\mathbb{R}$. Similarly, for the case of ω-securities, let $J^{\omega} = \left\{s \in \mathbb{R} \to \mathbb{R} \text{ such that } \int s\left(\omega\right) dF_{\omega}\left(\omega\right) = 0\right\}$. Finally, let $J = J^{\varepsilon} \cup J^{\omega}$ be the space of all securities.

Let $s^{\varepsilon}_{\ell,m} \in J^{\varepsilon}$ be the m-th insurance contract issued by issuer ℓ that is in principle contingent on all the idiosyncratic shocks of the economy. Similarly, $s^{\omega}_{\ell,m} \in J^{\omega}$ the corresponding insurance contract contingent on aggregate risk. Each contract can be represented as a function of the Arrow securities defined above. Let $p : J \to \mathbb{R}_{+}$ be the price schedule of these contracts. As it will be

[9] This assumption in the context of a model with endogenous creation of securities was first proposed by Pesendorfer (1995) (see Allen and Gale (1988), Allen and Gale (1991), Bisin (1998) for alternative assumptions).

[10] Makowski (1979) first derives the result that the price schedule in a competitive equilibrium with fixed costs of trading can be represented as a two-part tariffs.

clear after introducing the costs of trading securities, in equilibrium $p(\cdot)$ is not a linear function over J. To denote that agent i is not participating in the market for ε-risk (ω-risk) I will simply write $s_i^\varepsilon = \varnothing$ ($s_i^\omega = \varnothing$). I assume that only issuers can create and sell securities and, thus, a trade can occur only between a manager and an issuer. Marketing insurance contracts to potential buyers is a costly process. I assume that an issuer who sells an insurance contract conditional on ε-risk (ω-risk) has to pay a fixed cost $c_\varepsilon > 0$ ($c_\omega > 0$).

For simplicity, the trading costs are common across issuers and do not depend on the state where the security pays-off nor on the identity i of the project on which they are contingent. In other words, costs will differ only on whether the securities depend on idiosyncratic or aggregate risk. As it will be clear in following sections, an interesting comparative static exercise will be to vary the costs c_ε and c_ω and study the implications for the managers' investment decisions. This analysis will be central in section (1.7), where I introduce taxes to fix the inefficiency of the equilibrium. Indeed, in this model taxes rise the costs of issuing and selling securities and, thus, are isomorphic to a particular increase of the trading costs.

Payoffs. The fixed cost for every trade immediately implies that each manager will buy at most one insurance contract of each type (idiosyncratic or aggregate) from at most one issuer. This also implies that we can identify each insurance contract with the index of the manager i who buys it. Thus, $s_{\ell,i}^\varepsilon$ and $s_{\ell,i}^\omega$ will be the ε-contract and ω-contract created by issuer ℓ and customized to manager i, respectively. I will simply write $s_{\ell,i}$ to denote any insurance contract, idiosyncratic or aggregate, sold by issuer ℓ to manager i. Given that in equilibrium issuers make zero profits, it is without loss of generality assume that each issuer will face an equal mass of measure $1/N$ of agents. Without loss of generality, I assume that issuer ℓ trades with all the managers with index in $[(\ell-1)/N, \ell/N]$.

The final profits generated by manager i who buys contracts $s_{\ell,i}^\varepsilon$ and $s_{\ell,i}^\omega$ from issuer ℓ are

$$\Pi_i^m = \pi_i + s_{\ell,i}^\varepsilon - p\left(s_{\ell,i}^\varepsilon\right) + s_{\ell,i}^\omega - p\left(s_{\ell,i}^\omega\right). \tag{1.12}$$

Similarly, the profits of issuer ℓ[11] are

$$\Pi_\ell^I = \int_{(\ell-1)/N}^{\ell/N} \left(p\left(s_{\ell,i}^\varepsilon\right) - s_{\ell,i}^\varepsilon\right) di - \frac{c_\varepsilon}{N}.$$

The issuer is owned by investors, so it maximizes $\mathbb{E}\left[m(\omega)\,\Pi_\ell^I\right]$. As already discussed above, the equilibrium price of an insurance contract will now contain a fee to cover the fixed cost. Since ω-contracts are not traded on equilibrium, all prices at which issuers do not want to trade ω-contracts can be consistent with the equilibrium. Here, I am going to select the equilibrium price that leaves

[11] To simplify notation, I am writing profits of issuers using the fact that ω-contracts will not be traded in equilibrium.

issuers indifferent between trading or not.

Lemma 6. *The equilibrium price of an insurance contract $s^{\varepsilon}_{\ell,i}$ ($s^{\omega}_{\ell,i}$) is given by a two-part tariff: $p^{\varepsilon} + \int \int s^{\varepsilon}_{\ell,i}(\{\hat{\varepsilon}_j\})\, d\Phi_{\varepsilon}(\hat{\varepsilon}_j)\, dj$ ($p^{\omega} + \int s^{\omega}_{\ell,i}(\hat{\omega})\, m(\hat{\omega})\, dF_{\omega}(\hat{\omega}) / \mathbb{E}[m(\omega)]$), where $p^{\varepsilon} = c_{\varepsilon}$ and $p^{\omega} = c_{\omega}$.*

Except for the costs of trading contracts, the model of this section resembles the particular cases studied in sections 1.5.1 and 1.5.2. In particular, it is still true that the principal wants the manager not to trade ω-contracts. Finally, I am going to assume that the cost of trading ε-contracts is small enough that it is optimal for the principal to pay the fee p^{ε} and have the agent trade ε-contracts.

Assume now that trading costs are zero (so that $c_{\varepsilon} = c_{\omega} = 0$) and, as we did in section 1.5.1, and assume that in equilibrium it is optimal that the agent buys full insurance. The principal wants the agent to trade only the ε-risk and observes the transactions made by the agent. However, the principal does not observe the type of insurance contract that the agent is trading, that is, whether this contract is contingent on ε-risk or ω-risk. The agent is constrained by the principal to make only one transaction, thus, the only feasible deviation is to stop trading ε-securities and trade only ω-contracts. This double-deviation cannot be detected by the principal who will still observe that only one transaction has occurred.

Formally, let $\bar{U}_{\varepsilon\omega}(k, p(\cdot))$ be the value of the double-deviation for the agent, that is,

$$\bar{U}_{\varepsilon\omega}(k, p(\cdot)) = \max_{\hat{k}, s^{\omega} \neq \varnothing} \int u(\xi(x,\omega))\, dF_{x|\omega}\left(x|\omega; k, \hat{k}, s^{\varepsilon} = \varnothing, s^{\omega}, p(\cdot)\right) dF_{\omega}(\omega),$$

where $F_{x|\omega}\left(x|\omega; k, \hat{k}, s^{\varepsilon} = \varnothing, s^{\omega}, p(\cdot)\right)$ is the conditional distribution of x when the agent trades only the ω-contract s^{ω}.

The contracting problem becomes:

$$\max_{\xi, k} \int m(\omega)\left(x + \bar{R} + r\,k + (1-k)\,\omega - p^{\varepsilon} - \xi(x,\omega)\right) d\Phi(x)\, dF_{\omega}(\omega) \tag{P(full)}$$

subject to:

$$\frac{\partial}{\partial \hat{k}} \int u(\xi(x,\omega))\, dF_{x|\omega}\left(x|\omega; k, \hat{k}, s^{\varepsilon} = -k\,\varepsilon, s^{\omega} = \varnothing\right) dF_{\omega}(\omega)\Big|_{\hat{k}=k} - C'(k) = 0, \tag{IC$_k$}$$

$$\bar{U}_{\varepsilon\omega}(k, p(\cdot)) \leq \bar{u}, \tag{IC$_{\varepsilon\omega}$}$$

$$\int u(\xi(x,\omega))\, d\Phi(x)\, dF_{\omega}(\omega) - C(k) = \bar{u}. \tag{IR}$$

Here, $F_{x|\omega}\left(x|\omega; k, \hat{k}, s^{\varepsilon} = -k\varepsilon, s^{\omega} = \varnothing\right)$ is the conditional distribution of x when the agent buys full insurance against the ε-risk and doesn't trade ω-contracts. The optimal contract is a combination of 1.8 and 1.11, so I will not repeat it here.

The next proposition contains the effects on equilibrium k and welfare of changing the trading costs of the two types of insurance contracts.

Proposition 8. *For low enough prices c_ε and c_ω, when both types of insurance contracts are traded, equilibrium k and welfare decrease with the cost of ε-contracts, c_ε, and increase with the cost of ω-contracts, c_ω.*

Proposition 8 shows that changing the price of the two types of securities has opposite effects on equilibrium volatility and welfare. These comparative static results extend the conclusions derived separately in sections 1.5.1 and 1.5.2 to the case with fixed costs of trading. These effects will be the source of the trade-off faced by the social planner, which I consider in section 1.7, who has the power to tax transactions in the securities markets.

1.7 Efficiency and optimal policy

1.7.1 Taxation

This section studies the efficiency properties of the equilibrium derived in section 1.6. Of course, whether the equilibrium is socially optimal will depend on the powers we grant to the social planner. In particular, different conclusions on the efficiency of the equilibrium – and, thus, different policy prescriptions – follow from different assumptions on the information available to the social planner. A stark way to see this is by going back to the intuition behind the welfare implications of cheaper ω-risk insurance in section 1.5.2. There, I proved that, since the principal could always replicate the market allocation, he could only suffer from trades contingent on the ω-risk. It follows immediately that a social planner, who maximizes the welfare of investors and who can observe the trades of the different securities, could easily improve on the equilibrium allocation by forbidding the trades of ω-contracts. For this reason, in what follows I will restrict the social planner's information set by assuming that he doesn't have access to more information than the representative investors.

The inefficiency of the equilibrium follows from the fact that investors fail to coordinate the contract they design for the managers with the incentives faced by the issuers. This coordination failure is related to the conclusions in agency problems with multiple agents and common principal (Holmstrom and Milgrom (1990), Itoh (1993), Mookherjee (1984)). In most of these models, both agents face an agency problem and can potentially interact with each other. The principal has to design the contract by taking into account this interaction. In this model, the two agents are the managers and the issuers who interact through the securities market. Issuers are owned by the principal and they don't face any agency problem. However, the principal fails to understand how their activity affects prices and, thus, the incentives of the managers. The planner, then, can restore efficiency by fixing this coordination problem.

In this section, I consider two separate cases. First, I show that if the planner can observe the total number of transactions in the economy, but investors cannot observe the trading activity of managers (as in section 1.3), then a small positive on transactions can increase welfare in the economy. This tax makes it harder for managers to deviate and trade ω-contracts. Secondly, I show that when investors can observe the transactions made by each manager (as in section 1.6), then they can do better than the social planner and the transaction tax is redundant.

Suppose for now that the planner cannot observe the total quantity of transactions, but the investors do not observe any trading activity made by the managers (as in the model of section 1.3).

Let $e(\tau)$ be the equilibrium for a given value of the tax τ[12] and $\mathcal{W}(e(\tau))$ the equilibrium welfare of the investors. It is immediate to derive that $e(\tau)$ resembles the equilibrium derived in section 1.5, except that the insurance fees are now $p^{\varepsilon} + \tau$ and $p^{\omega} + \tau$, $\tau > 0$. The social planner chooses τ so as to maximize welfare for investors subject to allocations and prices being an equilibrium. Formally, the social planner solves

$$\max_{\tau} \mathcal{W}(e(\tau))$$

For given τ, this is exactly the same contracting problem as in section 1.6.

Lemma 7. *A small enough positive tax τ increases welfare in the economy.*

When choosing τ, the social planner optimally weighs the benefits and the costs of changing insurance prices p^{ε} and p^{ω}. A higher p^{ε} makes it more profitable for the agent to deviate by refusing to trade in the ε-risk market[13]. On the other hand, a higher p^{ω} has the effect of lowering the value for the agent of trading ω-contracts and, hence, relaxes the contracting problem. When τ is small enough, the latter effect dominates since the former effect is only second order.

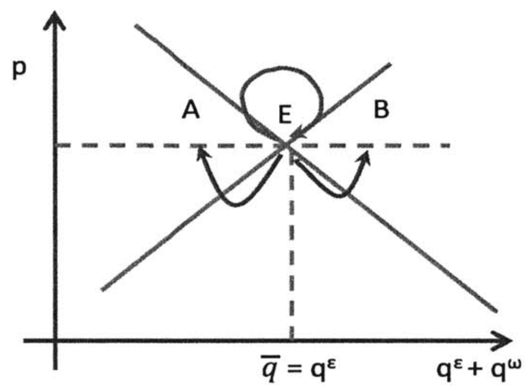
Suppose now that investors can observe the number of transactions made by the manager in the securities market as in section 1.6. The question is whether the planner can still improve on equilibrium welfare by using the transaction tax τ. A transaction is a trade between a manager and an issuer. We know that the principal wants the agent to trade only ε-contracts, thus, to execute only one transaction in the financial market. If the number of transactions is observable, then the only deviation for the manager that cannot be detected by the principal is when the manager stops trading ε-contracts and trades only ω-contracts (double-deviation).

To gain some intuition, suppose for a moment that only linear securities can be traded and there is a constant marginal cost for each unit of security. This case is easier to analyze since the choice variables are continuous. Suppose that the principal can observe the total number of units

[12] If we choose the price of ω-securities so that the fee p^{ω} equals the trading cost, the equilibrium is unique.

[13] Note that the tax raises revenues from the transactions of ε-securities (these securities are traded in equilibrium), but these revenues are rebated to investors.

bought by the manager, call it $\bar{q}$, but not the type of security traded. The principal can then use the extra choice variable $\bar{q}$ to control the trades of the managers. As usual, the principal will design a contract so that in equilibrium the manager will buy a quantity q^{ε} of ε-contracts and a quantity $q^{\omega} = 0$ of ω-contracts. Thus, the principal sets $\bar{q} = q^{\varepsilon}$. Now, when $\bar{q}$ is observable, at the margin the only deviation available to the manager is to reduce q^{ε} by dq^{ε} and increase q^{ω} by dq^{ω} =dq^{ε} so as to leave the total quantity $\bar{q}$ unchanged. On the contrary, when transactions are not observable, the manager has three possible deviations: decrease q^{ε}, increase q^{ω}, and do both. In the latter case, since the total quantity $\bar{q}$ is unchanged, a tax on transactions would have no effect on the value of the deviation. The following diagram illustrates the different possibilities: the first two deviations lead to points A and B, respectively, while the double-deviation leaves the total quantity of trades unaffected at point E.



Intuitively, the constraint on the quantity traded is at least weakly preferred to a tax on the transactions in the setting with linear securities. In ongoing work, I am exploring the consequences of restricting the space of securities to linear securities, but to allow investors to observe a signal on the total amount of resources that managers invest in the trading activity. However, managers can still benefit from a deviation that leaves the value of trades unaffected. The interpretation is that investors have access to the balance sheets of financial institutions and can infer the amount of resources spent in trading activities. However, they don't have the expertise to understand the type of securities that are being traded.

A similar result holds in the case considered here as the following proposition shows.

Proposition 9. *When transactions are observable, the transaction tax cannot improve on equilibrium welfare.*

1.7.2 Regulation

When transactions are observable the planner cannot help investors by taxing them. However, managers have still access to a double-deviation that allows them to trade insurance contracts contingent on ω. Thus, ω-contracts can still be traded off the equilibrium.

Taxing transactions in derivatives markets is not the only possible way to increase welfare in this economy. Thus, that even maintaining the assumption that the social planner has no superior information over the other agents, the social planner can do much better by *regulating* the issuers of financial securities. By regulation I mean giving the planner the power to write a contract that incentivizes issuers to maximize welfare in the economy. Regulation goes to the heart of the coordination problem of the investors: the planner takes the place of investors and realizes that incentives for managers have to be coordinated with incentives for issuers.

To see this, remember that trades of securities contingent on the aggregate risk can only undo the incentives set up by investors and these securities are not traded in equilibrium. As shown in the analysis of section 1.5.2, the presence of issuers selling ω-contracts matters to the extent that it constrains the contracts space of the principal. In other words, ω-contracts matter only as they can represent a profitable deviation for the agents. Hence, issuers trade only ε-contracts and make constant (zero) profits in equilibrium. In contrast, off the equilibrium, issuers sell ω-contracts to managers and, thus, take some aggregate risk on their balance sheets.

Profits of issuers are assumed to be observable, hence, a social planner can always increase welfare in the economy by punishing any volatility of these profits. Therefore, the optimal regulation in this model is to forbid issuers to ever take aggregate risk on their balance-sheets and to punish them in case of deviation. This policy limits (and, in the extreme, eliminates) the incentives to trade aggregate risk out of the equilibrium and, therefore, it relaxes the investors' problem.

Formally, I am going to assume that the social planner can regulate the trading activity of financial issuers by choosing a function $\eta_\ell : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ that maps pairs $\left(\Pi_\ell^I, \omega\right)$ into a payment to each issuer.

Proposition 10. *Let Π_ℓ^{I*} be the equilibrium profits of issuer ℓ (Π_ℓ^{I*} is zero since ω-contracts are not traded in equilibrium). Then, the optimal policy η_ℓ^* is given by*

$$\eta_\ell^* \left(\Pi_\ell^{I*}, \omega\right) = \Pi_\ell^{I*} \text{ and } \eta_\ell^* \left(\Pi_\ell^{I}, \omega\right) = -\infty \text{ if } \Pi_\ell^{I} \neq \Pi_\ell^{I*}.$$

If this policy is implemented then equilibrium welfare coincides with that of proposition 5.

The intuition for proposition 10 has already been discussed. The result on welfare is also quite intuitive. If the optimal policy η_ℓ^* is implemented, then issuers will never find it optimal to sell

ω-contracts. Thus, the IC constraint that precludes deviations with ω-contracts drops from the contracting problem and we are back to the case of section 1.5.1.

The reason why regulation is so effective is not related to the fact that ω-contracts are not traded in equilibrium and profits of issuers are constant. If there was some trading of aggregate risk in equilibrium (say, due to some unmodelled demand for hedging), then profits would vary with the aggregate state of the economy. However, as long as the planner can determine the right amount of aggregate risk that issuers can hold on their balance sheets, then the optimal policy would still have the same power as in proposition 10. Of course, this is possible because the social planner is assumed to observe the aggregate state ω. The optimal regulation is then given by a limit on how risky the balance sheets of the issuers of financial securities can be.

The optimal policy still requires a great deal of information to be implemented since the social planner has to understand what is the optimal amount of aggregate risk that should be traded. In particular, the planner has to realize what part of the ω-risk is traded by those portfolio managers who determine the quantity of risk in the economy through their investment decisions. While this is easy in this abstract model, it may be less so in real financial markets.

1.8 Discussion

Unlike the securities of this model – which are contingent either on idiosyncratic or aggregate risk – derivative contracts traded in financial markets are often contingent on many different risks, both aggregate and idiosyncratic. Thus, while the model highlights a fundamental difference between securities contingent on idiosyncratic and aggregate risks, this distinction is much less clear in real financial markets.

Nonetheless, we can argue that some financial securities are more sensitive to idiosyncratic risks while others are used to hedge risks that are more aggregate. For example, a Credit Default Swap (CDS)[14] that insures the buyer against the default of a firm that is independent from the rest of the economy is a derivative that is relatively more sensitive to idiosyncratic risks. On the contrary, a CDS written on a bond issued by a big firm (say, GE or Walmart) is likely to be relatively more sensitive to the aggregate state of the economy.

Another example of a security contingent on aggregate risk is an Interest Rate Swap that banks use to hedge the interest rate risk of their loan portfolios.

Finally, a more involved example is given by a tranche of a Funded Synthetic CDO[15]. This

[14]A CDS is a credit derivative which obliges the seller to compensate the buyer in the event of a loan default. The buyer pays a premium to the seller for this insurance.

[15]This is a derivative contract that allows investors with different appetite for risk buy tranches of a Special Purpose Vehicle (SPV). The SPV then buys Treasuries and sells a portfolio of CDS. The buyers of the CDS pay a periodic premium to the SPV which transfers it to the investors. However, if a loan in the portfolio defaults, then the Treasuries are sold to pay the buyer of the protection.

derivative allows an investors to take a position on the credit risk of a portfolio of loans and, as the number of loans in the underlying portfolio increases, idiosyncratic risks wash away and the CDO will be relatively more sensitive to common risks.

Recent work in empirical finance focuses on how trading in derivatives markets affects the risk of financial institutions. Ideally, to see whether the predictions of the model are consistent with the data, we would need to make a distinction between securities contingent on idiosyncratic or aggregate risks. In the former case, the model predicts that banks' balance sheets become less correlated with each other while the latter case leads to the opposite conclusion. This ideal experiment assumes that we can distinguish financial securities depending on the type of risk they hedge. In practice, however, this distinction is not as sharp.

A less demanding exercise would be to ask what happens to a financial institution's balance sheet after it start trading in the derivatives markets. In a recent work, Nijskens and Wagner (2011) study two separate datasets of banks which include information on various types of securitization around the world. In particular, one dataset contains data on Credit Default Swaps (CDS) and the other on Collateralized Loan Obligation (CLO)[16].

Both datasets allow Nijskens and Wagner (2011) to observe the date on which each bank start trading each of these financial products. They then look at the effect on the returns of each bank after the date of the first trade of CDS or CLO. They find that a bank that trades CDS or CLO experiences a permanent *increase* in its beta, which is a measure of the systematic risk of a bank. Also, the magnitude of such effect is bigger in the case of CLO. Remember that in our model the profits of a bank are given by

$$\pi_i = \bar{R} + (r + \varepsilon_i) k_i + \omega (1 - k_i) + u_i.$$

If we take the average across different sectors, $\int \pi_i \, di$, then the market return is given by $\bar{R}+\omega(1-k)$ (where k is the equilibrium choice of all managers), If we let σ_i be the standard deviation of the return of bank i, the correlation of bank i with the market is $\rho_i = (1-k)\sigma_\omega/\sigma_i$. Nijskens and Wagner (2011) find that, after the first CLO or CDS trade, the value of ρ_i increases while the relative volatility $\sigma_i/(1-k)\sigma_\omega$ decreases. This is consistent with the prediction of this model that derivatives tend to decrease the value of k_i. It is also tempting to speculate that the bigger effect of CLO trades on the beta of the bank relative to CDS trades is related to the fact that CLOs, which are pools of loans, are more similar to the ω-securities of this paper.

Similar evidence is found by Haensel and Krahnen (2007). They use a dataset of CDOs issued by European financial institutions. They also find that banks engaging in these transactions tend to increase their exposure to the market. Of course, while these results are consistent with the conclusions of this paper, they are certainly not conclusive evidence. There may be many reasons

[16] A CLO is a form of securitization through which banks transfer pools of loans to the buyers of these securities. The payoff of this derivative resembles, to a first approximation, the payoff of a funded synthetic CDO.

for why banks increase their systematic risk after trading some types of derivatives.

On the information side, this model implies that an easy way to improve welfare is by requiring more information disclosure. Formally, this would be equivalent to modify part (d) of Assumption 1 and assume perfect observability of trades and types of security. Once investors have the ability to contract on the different securities traded by managers, they will forbid trading of ω-risk (and allow trading of ε-risk). In fact, we can conjecture that the equilibrium when part (d) of assumption 1 is removed will resemble that of section 1.5.1.

While information disclosure is a strong and interesting implication of this model, it derives from the mathematical way I chose to model complex securities and OTC markets. In general, it is realistic to assume that even if big financial institutions were required to disclose all their trading activities to outside investors, it would probably be a daunting task for many investors to process this information (Brunnermeier and Oehmke (2011)).

For simplicity, in this model I have assumed that agents and issuers trade in a Walrasian market by paying a fixed cost per trade. These costs can be interpreted as a reduced-form way to capture the effects of imperfect competition in the securities market or the liquidity of these markets. Remember that the financial market in this model is an abstraction of OTC markets where typically market makers provide liquidity by posting a price and trading securities at that price. The creation of new financial products and the growth of OTC markets have stimulated important research on decentralized markets. These papers explore the main features of these markets like price determination, liquidity and diffusion of information. Duffie et al. (2005), for example, provide a theory of asset pricing in decentralized markets[17]. The focus of this paper, however, is not about the specific trading environment, but on how complex securities can affect the portfolio choice of investors and, thus, the aggregate volatility of the economy.

A Walrasian market for securities is also the typical assumption in the literature on markets with endogenous securities creation (Allen and Gale (1991), Pesendorfer (1995), Bisin (1998)). These papers depart from the standard assumption that traded securities are exogenously given and, instead, assume that they are issued by optimizing issuers. Some of these papers, in particular, assume that issuers have market power (Allen and Gale (1988) and Bisin (1998)). This alternative assumption is not explored in this paper, but from the social planner's problem we can conjecture that, by increasing the price of the insurance contracts contingent on ω, some market power may actually be beneficial for welfare. Similarly, if we interpret the trading cost as the liquidity of these markets, then it may be that case that *less liquid* markets are beneficial for welfare.

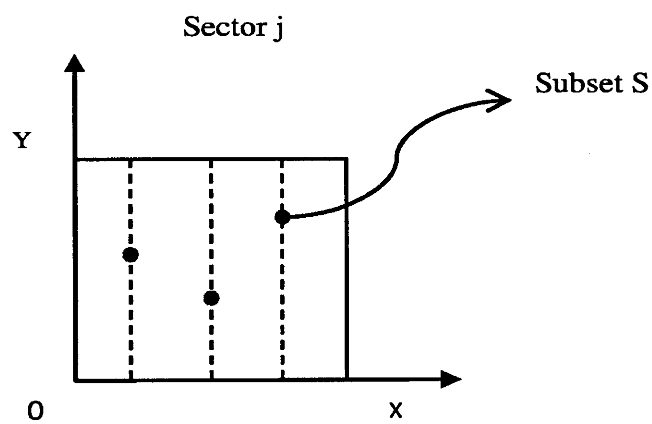
The stark conclusion about the welfare effects of ω-securities depend on some strong assumptions of the model. First, the assumption of symmetric preferences, technology, and equilibrium eliminates any gains from trading aggregate risk among managers. Also, I have assumed that investors can

[17]Other important contributions are Duffie et al. (2005), Duffie et al. (2007), Duffie and Manso (2007), Lagos (2010), Lagos and Rocheteau (2009), Lagos et al. (2007), Vayanos (1998), Vayanos and Weill (2008), and Weill (2008).

perfectly condition their contracts on aggregate states, but they cannot do the same for idiosyncratic states. Financial markets help allocate aggregate risk to the agents who are better prepared to hold it. However, as long as investors cannot fully control the risks traded by their managers, then trading of ω-risk has the potential to reduce welfare. Also, the assumptions of this model help me isolate this particular mechanism and analyze its (negative) implications. In a more general model, different effects of aggregate risk trading would coexist and the optimal policy would be characterized by a richer set of actions.

1.9 Appendix 1. Project Selection

In this appendix, I model the project selection process of section 1.3 more explicitly. In the economy there is a continuum of sectors denoted by $j \in [0,1]$. In each sector j there is a two-dimensional continuum of potential projects indexed by $(x, y) \in [0,1]^2$. A one-dimensional subset $S \subset [0,1]^2$ of these projects are specialized projects, the rest are standard projects. In particular, for each $x \in [0,1]$ there is a unique y such that $(x, y) \in S$. This value of y is randomly drawn from a uniform distribution on $[0,1]$ for each x. The set of projects is depicted in the figure below. Every manager is an expert of a particular sector and has access to information about projects in that sector. More precisely, if a manager i is an expert of sector j, then he can exert one unit of effort and inspect all the projects with a given index x in his sector and find out the specialized project $(x, y) \in S$. If no effort is exerted, the manager has zero probability of choosing the specialized project.



All the specialized projects in sector j have the same random payoff $r_{i,j} = \bar{r} + \varepsilon_j + u_i$ and all the standard projects have the same random payoff $R_i = \bar{R} + \omega + u_i$. Thus, a manager who wants to invest an amount k in specialized projects has to exert k units of effort and suffer a loss of utility equal to $C(k)$.

1.10 Appendix 2. Proofs and additional lemmas

Proofs and additional lemmas

Proof of proposition 1. Assume for now that the FOA is valid, the maximization problem P(NS) is

$$\max_{\xi,k} \int m(\omega) \left(x + \bar{R} + rk + (1-k)\omega - \xi(x,\omega)\right) d\Phi(x)\, dF_\omega(\omega)$$

subject to:

$$\frac{\partial}{\partial \hat{k}} \int u\left(\xi\left(x,\omega\right)\right) dF_{x|\omega}\left(x|\omega;k,\hat{k}\right) dF_{\omega}\left(\omega\right)\Big|_{\hat{k}=k} - C'\left(k\right) = 0,$$

$$\int u\left(\xi\left(x,\omega\right)\right) d\Phi\left(x\right) dF_{\omega}\left(\omega\right) - C\left(k\right) \geq 0.$$

By using the properties of Gaussian distributions together with the moments (1.3), I can rewrite the IC constraint as:

$$\int u\left(\xi\left(x,\omega\right)\right) \left(\frac{1}{\sigma_x}\left(r-\omega\right)x + \frac{k\,\sigma_\varepsilon^2}{\sigma_x^2}\left(x^2-1\right)\right) d\Phi\left(x\right) dF_{\omega}\left(\omega\right) - C'\left(k\right) = 0.$$

Here, I have used the fact that in equilibrium x and ω are independent by construction. Let λ and μ be the Lagrange multipliers on the two constraints, respectively, and define the Lagrangian:

$$\begin{aligned}
\Lambda \;=\; & \int m\left(\omega\right)\left(x+\bar{R}+rk+\left(1-k\right)\omega-\xi\left(x,\omega\right)\right) d\Phi\left(x\right) dF_{\omega}\left(\omega\right) \\
& +\lambda\left(\int u\left(\xi\left(x,\omega\right)\right)\left(\frac{1}{\sigma_x}\left(r-\omega\right)x+\frac{k\,\sigma_\varepsilon^2}{\sigma_x^2}\left(x^2-1\right)\right) d\Phi\left(x\right) dF_{\omega}\left(\omega\right) - C'\left(k\right)\right) \\
& +\mu\left(\int u\left(\xi\left(x,\omega\right)\right) d\Phi\left(x\right) dF_{\omega}\left(\omega\right) - C\left(k\right)\right)
\end{aligned}$$

If we differentiate Λ pointwise w.r.t. $\xi\left(x,\omega\right)$, we get that the optimal contract solves:

$$\frac{m\left(\omega\right)}{V'\left(\xi\left(x,\omega\right)\right)} = \mu + \lambda\left[\frac{1}{\sigma_x}\left(r-\omega\right)x + \frac{k\,\sigma_\varepsilon^2}{\sigma_x^2}\left(x^2-1\right)\right]$$

which is the expression in proposition 1. □

Proof of proposition 3. To see that this equilibrium is efficient we can set up the social planner's problem. The only difference between the planner's problem and the equilibrium is that the former takes into account how aggregate consumption $c\left(\omega\right)$ depends on the profits of all the managers. Formally, the planner solves:

$$\max_{\xi,k,c} \int v\left(c\left(\omega\right)\right) d\Phi\left(x\right) dF_{\omega}\left(\omega\right)$$

subject to:

$$\frac{\partial}{\partial \hat{k}} \int u\left(\xi\left(x,\omega\right)\right) dF_{x|\omega}\left(x|\omega;k,\hat{k}\right) dF_{\omega}\left(\omega\right)\Big|_{\hat{k}=k} - C'\left(k\right) = 0,$$

$$\int u\left(\xi\left(x,\omega\right)\right) d\Phi\left(x\right) dF_{\omega}\left(\omega\right) - C\left(k\right) \geq 0,$$

$$c\left(\omega\right) + \int \xi\left(x,\omega\right) d\Phi\left(x\right) = \int \left[x+\bar{R}+r\,k+\left(1-k\right)\omega\right] d\Phi\left(x\right), \;\forall\omega.$$

The last constraint is the resource constraint of the economy. Note that only this constraint depends on $c(\omega)$. So, if we let $\varphi(\omega) f_\omega(\omega)$ denote the Lagrange multilpier on the resource constraint, we can separate the problem by first solving for c:

$$\max_c \int v(c(\omega))\, d\Phi(x)\, dF_\omega(\omega)$$

subject to:

$$c(\omega) + \int \xi(x,\omega)\, d\Phi(x) = \int \left[x + \bar{R} + r\ k + (1-k)\,\omega\right] d\Phi(x), \ \forall \omega.$$

Setting up the Lagrangian and taking the first-order condition w.r.t. $c(\omega)$ gives:

$$m(\omega) \equiv v'(c(\omega)) = \varphi(\omega).$$

Now, for given $\varphi(\omega)$, we can solve the dual problem of maximizing total resources, that is,

$$\max_{\xi,k} \int m(\omega) \left[x + \bar{R} + r\ k + (1-k)\,\omega - \xi(x,\omega)\right] d\Phi(x)\, dF_\omega(\omega),$$

subject to the IR and IC constraint. This is the same problem as P(NS). This proves that the equilibrium is a solution to the planner's problem. □

1.10.1 Only ε-securities

Proof of lemma 1. Consider the portfolio problem of the representative investor:

$$\max_y \mathbb{E}\left[m(\omega)\,\Pi^I\right],$$

where

$$\Pi^I = \int (p_{i,\hat{\varepsilon}} - z_{i,\hat{\varepsilon}})\, y_{i,\hat{\varepsilon}}\ d\hat{\varepsilon}\ di.$$

Take the first-order condition pointwise w.r.t. y

$$p_{i,\hat{\varepsilon}} - \phi_\varepsilon(\hat{\varepsilon}) \geq 0.$$

If $p_{i,\hat{\varepsilon}}$ was different from $\phi_\varepsilon(\hat{\varepsilon})$, then the investor would buy or sell an infinite amount of this security. This cannot be an equilibrium, so $p_{i,\hat{\varepsilon}} = \phi_\varepsilon(\hat{\varepsilon})$. □

Proof of lemma 2. The optimality of full insurance can be proved using the results in [illegible] ([illegible]) and [illegible] ([illegible]).

These papers show how to rank different information systems in the principal-agent framework. They consider a principal-agent model in which the principal can choose among different set of signals about the agent's action. An information system will be preferred to another if the former can implement every action at a lower cost for the principal. This criterion is more general than the informativeness criterion in Holmstrom (1979) who considers only information systems which are inclusive in the sense that one system contains more signals than the other (Holmstrom (1979) defines the notion of informativeness in terms of a sufficiency criterion).

This is, however, restrictive in our context since systems are not inclusive. Kim (1995) and Jewitt (2007) show how to rank information systems which are not inclusive in terms of their likelihood ratios. More specifically, they show that, given two information systems, one of them will implement any action at a lower cost for the principal if and only if its likelihood ratio is a mean preserving spread of the other.

The result in Kim (1995) requires the principal to be risk-neutral, but in this paper there is the stochastic discount factor $m(\omega)$. However, the argment easily generalizes to our case if we consider the *conditional* likelihood ratio. Thus, I will use the result that given two signals $\hat{x}$ and $\tilde{x}$, it is cheaper for a principal to implement a given action k with $\hat{x}$ if the conditional likelihood ratio

$$L_{\hat{x}|\omega}\left(x|\omega;k,d^{\varepsilon},p\left(\cdot\right)\right)=\frac{\partial f_{\hat{x}|\omega}\left(x|\omega;k,k,d^{\varepsilon},p\left(\cdot\right)\right)/\partial\hat{k}}{f_{\hat{x}|\omega}\left(x|\omega;k,k,d^{\varepsilon},p\left(\cdot\right)\right)}$$

is riskier than the likelihood ratio obtained with $\tilde{x}$. Intuitively, since different actions lead to different realizations of the signal with higher probability, the higher the volatility of the likelihood ratio, the more informative the signal.

Also, when $\hat{k}=k$ the distribution $F_{x|\omega}$ does not depend on ω, so I will simply write F_x. The mean of $L_{x|\omega}$ is given by:

$$\begin{aligned}
&\int\left(L_{x|\omega}\left(x|\omega;k,d^{\varepsilon},p\left(\cdot\right)\right)\right)dF_x\left(x;k,d^{\varepsilon},p\left(\cdot\right)\right)dF_{\omega}\left(\omega\right)\\
=&\int\frac{\partial}{\partial\hat{k}}\int f_{x|\omega,\varepsilon}\left(x|\omega;k,d^{\varepsilon},p\left(\cdot\right)\right)\;d\Phi_{\varepsilon}\left(\varepsilon\right)\;dx\\
=&\int\int\left(\frac{\left(r-\omega+\varepsilon\right)\left(x-k\;\varepsilon\right)}{\sigma_u^2}\right)f_{x|\omega,\varepsilon}\left(x|\omega,\varepsilon;k,d^{\varepsilon},p\left(\cdot\right)\right)\;d\Phi_{\varepsilon}\left(\varepsilon\right)\;dx\\
=&\int\int\left(\frac{\left(r-\omega+\varepsilon\right)\left(x-k\;\varepsilon\right)}{\sigma_u^2}\right)dF_{\varepsilon|\omega,x}\left(\varepsilon|\omega,x;k,d^{\varepsilon},p\left(\cdot\right)\right)\;dF_x\left(x;k,d^{\varepsilon},p\left(\cdot\right)\right)\\
=&\frac{1}{\sigma_u^2}\int\left(\left(r-\omega+\mathbb{E}\left[\varepsilon|\omega,x\right]\right)x-k\mathbb{E}\left[\left(r-\omega+\varepsilon\right)\varepsilon|\omega,x\right]\right)dF_x\left(x;k,d^{\varepsilon},p\left(\cdot\right)\right)\\
=&\frac{1}{\left(\sigma_u^2\right)^2}\int\left(\left(r-\omega\right)\mathbb{E}\left[u|x\right]+\mathbb{E}\left[\varepsilon\;u|x\right]\right)dF_x\left(x;k,d^{\varepsilon},p\left(\cdot\right)\right)=0,
\end{aligned}$$

since the distribution of x does not depend on ω.

Thus, the variance of $L_{x|\omega}$ is:

$$
\begin{aligned}
& \int \left(L_{x|\omega}\left(x|\omega;k,d^{\varepsilon},p\left(\cdot\right)\right)\right)^{2} dF_{x}\left(x;k,d^{\varepsilon},p\left(\cdot\right)\right) dF_{\omega}\left(\omega\right) \\
= & \int \frac{1}{f_{x}\left(x;k,d^{\varepsilon},p\left(\cdot\right)\right)}\left(\frac{\partial}{\partial \hat{k}}\int f_{x|\omega,\varepsilon}\left(x;k,d^{\varepsilon},p\left(\cdot\right)\right) d\Phi_{\varepsilon}\left(\varepsilon\right)\right)^{2} dx \\
= & \int \frac{1}{f_{x}\left(x;k,d^{\varepsilon},p\left(\cdot\right)\right)}\left(\int\left[\frac{\left(r-\omega+\varepsilon\right)\left(x-k\ \varepsilon\right)}{\sigma_{u}^{2}}\right] f_{x|\omega,\varepsilon}\left(x|\omega,\varepsilon;k,d^{\varepsilon},p\left(\cdot\right)\right) d\Phi_{\varepsilon}\left(\varepsilon\right)\right)^{2} dx \\
= & \int\left(\int\left(\frac{\left(r-\omega+\varepsilon\right)\left(x-k\ \varepsilon\right)}{\sigma_{u}^{2}}\right) dF_{\varepsilon|\omega,x}\left(\varepsilon|\omega,x;k,d^{\varepsilon},p\left(\cdot\right)\right)\right)^{2} dF_{x}\left(x;k,d^{\varepsilon},p\left(\cdot\right)\right) \\
= & \frac{1}{\left(\sigma_{u}^{2}\right)^{2}}\int\left(\left(r-\omega+\mathbb{E}\left[\varepsilon|\omega,x\right]\right)x-k\mathbb{E}\left[\left(r-\omega+\varepsilon\right)\varepsilon|\omega,x\right]\right)^{2} dF_{x}\left(x;k,d^{\varepsilon},p\left(\cdot\right)\right) \\
= & \frac{1}{\left(\sigma_{u}^{2}\right)^{2}}\int\left(\left(r-\omega\right)\mathbb{E}\left[u|x\right]+\mathbb{E}\left[\varepsilon\ u|x\right]\right)^{2} dF_{x}\left(x;k,d^{\varepsilon},p\left(\cdot\right)\right)
\end{aligned}
$$

Now, for given distribution for ε, if the variance σ_{ε}^{2} of this distribution increases, then $\mathbb{E}\left[u|x\right] \to 0$ $\forall x$ (x becomes a worse predictor for u). On the contrary, as σ_{ε}^{2} increases, the cross moment $\mathbb{E}\left[\varepsilon\ u|x\right]$ also increases.

The condition for full insurance to be optimal is that the first effect dominates the second. Formally, the function

$$G\left(\omega,d^{\varepsilon},k\right)=\frac{1}{\left(\sigma_{u}^{2}\right)^{2}}\int\left(\left(r-\omega\right)\mathbb{E}\left[u|x\right]+\mathbb{E}\left[\varepsilon\ u|x\right]\right)^{2} dF_{x}\left(x;k,d^{\varepsilon},p\left(\cdot\right)\right)$$

is maximized at $d^{\varepsilon}=-k\ \varepsilon$ for any value of ω and k. This condition is implicit because different choices of d^{ε} affect the distribution of x.

However, if we restrict attention to linear securities, then x follows a Gaussian distribution and, in particular,

$$\mathbb{E}\left[\varepsilon|x\right]=\frac{k\sigma_{\varepsilon}^{2}}{\sigma_{x}^{2}}x,$$

and

$$\mathbb{E}\left[\varepsilon^{2}|x\right]=\frac{k\sigma_{\varepsilon}^{2}}{\sigma_{x}^{2}}x=\frac{\sigma_{u}^{2}\sigma_{\varepsilon}^{2}}{\sigma_{x}^{2}}+\left(\frac{k\sigma_{\varepsilon}^{2}}{\sigma_{x}^{2}}\right)^{2}x^{2}.$$

Therefore,

$$\int\left(L_{x|\omega}\left(x|\omega;k,d^{\varepsilon}\right)\right)^{2} dF_{x}\left(x;k,d^{\varepsilon}\right)=\frac{\left(r-\omega\right)^{2}}{\sigma_{x}^{2}}+\frac{k^{2}\left(\sigma_{\varepsilon}^{2}\right)^{2}}{\left(\sigma_{x}^{2}\right)^{2}}.$$

A sufficient condition for this expression to be maximized at $\sigma_{\varepsilon}^{2}=0$ for every value of ω and k is: $0\leq\sigma_{u}^{2}\leq\left(r-\omega\right)^{2},\forall\omega$. This is the condition in lemma 2. □

Proof of proposition 1.8. Assume that he conditions for full insurance to be optimal are met. Under the assumption that the FOA is valid, we can rewrite $\mathrm{P}(\varepsilon)$ assuming that the distribution of ε is degenerate at 0. By taking the first-order condition w.r.t. ξ, we obtain:

$$\frac{m(\omega)}{V'(\xi(x,\omega))} = \mu + \lambda \frac{1}{\sigma_x}(r-\omega)x.$$

With this contract the problem of the manager is concave in x. This implies two things. First, the agent will buy full insurance at the actuarially fair price of lemma 1. In turn, this means that this contract is optimal for the principal. Secondly, the concavity of the problem implies that the FOA is valid (Jewitt (1988)). $\square$

Proof of proposition 5. Let $C^*(k)$ and $C^*_\varepsilon(k, p(\cdot))$ be the minimum costs of implementing k when no securities are available and when the agent fully hedges his idiosyncratic risk, respectively. I consider the equilibrium where $p(\cdot)$ is given by lemma 1. To prove that the optimal k increases when the agent fully insures his risk, I can show that $C^*(k) - C^*_\varepsilon(k, p(\cdot))$ is increasing is k and use a monotone comparative static argument. First, note that under the assumption that full insurance is optimal, $C^*(k) - C^*_\varepsilon(k, p(\cdot)) > 0$.

Remember that in equilibrium the utility of the agent is an average of $\tilde{u}\left((\mu + \lambda L_{x|\omega}(x|\omega; k, d^\varepsilon, p(\cdot)))/m(\omega)\right)$. By the envelope theorem, differentiating $C^*(k) - C^*_\varepsilon(k, p(\cdot))$ w.r.t. k gives:

$$\begin{aligned}
& \frac{\partial}{\partial k}\left(C^*(k) - C^*_\varepsilon(k, p(\cdot))\right) \\
= \; & -\hat{\lambda}\int \tilde{u}\left(\frac{\hat{\mu} + \hat{\lambda}L_{x|\omega}(x|\omega; k, 0, p(\cdot))}{m(\omega)}\right)\frac{\partial}{\partial k}L_{x|\omega}(x|\omega; k, 0, p(\cdot))\, d\Phi(x)\, dF_\omega(\omega) + \hat{\lambda}C''(k) \\
& +\lambda\int \tilde{u}\left(\frac{\hat{\mu} + \hat{\lambda}L_{x|\omega}(x|\omega; k, -k\,\varepsilon, p(\cdot))}{m(\omega)}\right)\frac{\partial}{\partial k}L_{x|\omega}(x|\omega; k, -k\,\varepsilon, p(\cdot))\, d\Phi(x)\, dF_\omega(\omega) - \lambda C''(k) \\
& -\frac{\partial}{\partial k}\hat{\mu}\int \tilde{u}\left(\frac{\hat{\mu} + \hat{\lambda}L_{x|\omega}(x|\omega; k, 0, p(\cdot))}{m(\omega)}\right) d\Phi(x)\, dF_\omega(\omega) + \hat{\mu}C'(k) \\
& +\frac{\partial}{\partial k}\mu\int \tilde{u}\left(\frac{\hat{\mu} + \hat{\lambda}L_{x|\omega}(x|\omega; k, -k\,\varepsilon, p(\cdot))}{m(\omega)}\right) d\Phi(x)\, dF_\omega(\omega) - \mu C'(k),
\end{aligned}$$

where a "hat" denotes the Lagrange multipliers for the case with no insurance. Therefore,

$$\begin{aligned}
\frac{\partial}{\partial k}\left(C^*(k) - C^*_\varepsilon(k, p(\cdot))\right) &= -\hat{\lambda}\int \tilde{u}\left(\frac{\hat{\mu} + \hat{\lambda}L_{x|\omega}(x|\omega; k, 0, p(\cdot))}{m(\omega)}\right)\frac{\partial}{\partial k}L_{x|\omega}(x|\omega; k, 0, p(\cdot))\, d\Phi(x)\, dF_\omega(\omega) \\
&\quad + \left(\hat{\lambda} - \lambda\right)C''(k) + (\hat{\mu} - \mu)C'(k) \\
&= -\hat{\lambda}\int \tilde{u}\left(\frac{\hat{\mu} + \hat{\lambda}L_{x|\omega}(x|\omega; k, 0, p(\cdot))}{m(\omega)}\right)\frac{\partial}{\partial k}L_{x|\omega}(x|\omega; k, 0, p(\cdot))\, d\Phi(x)\, dF_\omega(\omega) + \\
&\quad \underbrace{\left(\hat{\lambda} - \lambda\right)}_{>0}C''(k) + \underbrace{(\hat{\mu} - \mu)}_{>0}C'(k) > 0,
\end{aligned}$$

where $\hat{\lambda} > \lambda$ and $\hat{\mu} > \mu$ are implied by the assumption that insuring the idiosyncratic risk is optimal for the principal and $\int \tilde{u}\left((\hat{\mu} + \hat{\lambda} L_{x|\omega}(x|\omega; k, 0, p(\cdot)))/m(\omega)\right) \frac{\partial}{\partial k} L_{x|\omega}(x|\omega; k, 0, p(\cdot))\, d\Phi(x)\, dF_\omega(\omega) < 0$ since the variance of $L_{x|\omega}$ is lower when k is higher.

The proof that welfare is higher when ε-securities are traded follows from similar steps to the proof of proposition 3. I first define a social planner that chooses ξ, k, and c so as to maximize the welfare of investors. Then I show that, for given aggregate consumption, the planner's problem is equivalent to finding the optimal contract that maximizes the value of resources produced by each single manager. Under the assumption that full insurance is optimal, it follows that the value of resources when ε-securities are available is higher. This proves the claim that welfare in the economy is higher. □

1.10.2 Only ω-securities

Proof of lemma 4. Again, consider the portfolio problem of the representative investor:

$$\max_y \mathbb{E}\left[m(\omega)\, \Pi^I\right],$$

where

$$\Pi^I = \int (p_{\hat{\omega}} - z_{\hat{\omega}})\, y_{\hat{\omega}}\, d\hat{\omega},$$

and differentiate the expression pointwise w.r.t. y

$$\mathbb{E}\left[m(\omega)\right] p_{\hat{\omega}} - m(\hat{\omega})\, f_\omega(\hat{\omega}) \geq 0.$$

If $p_{\hat{\omega}}$ was different from $m(\hat{\omega})\, f_\omega(\hat{\omega}) / \mathbb{E}\left[m(\omega)\right]$, then the representative investor would buy or sell an infinite amount of this security. This cannot be an equilibrium, so $p_{\hat{\omega}} = m(\hat{\omega})\, f_\omega(\hat{\omega}) / \mathbb{E}\left[m(\omega)\right]$. □

Proof of lemma 1.11. When only ω-securities are traded, the optimal contract solves

$$\max_{\xi,k} \int m(\omega)\left(x + \bar{R} + r\, k + (1-k)\,\omega - \xi(x,\omega)\right) d\Phi(x)\, dF_\omega(\omega)$$

subject to:

$$\frac{\partial}{\partial \hat{k}} \int u(\xi(x,\omega))\, dF_{x|\omega}\left(x|\omega; k, \hat{k}, 0, p(\cdot)\right) dF_\omega(\omega)\Big|_{\hat{k}=k} - C'(k) = 0,$$

$$\frac{\partial}{\partial \hat{d}} \int u(\xi(x,\omega))\, dF_{x|\omega}\left(x|\omega; k, k, \hat{d}, p(\cdot)\right) dF_\omega(\omega)\Big|_{\hat{d}=0} = 0,$$

$$\int u(\xi(x,\omega))\, d\Phi(x)\, dF_\omega(\omega) - C(k) = \bar{u}.$$

This is program P(ω) in the main text. I am assuming here that the FOA is valid. We can now define the Lagrangian and maximize it poitwise w.r.t. ξ. The optimal contract solves:

$$\frac{m(\omega)}{V'(\xi(x,\omega))} = \mu + \lambda \frac{1}{\sigma_x}(r - h(\omega))\ x + \lambda \frac{k\,\sigma_\varepsilon^2}{\sigma_x^2}\left(x^2 - 1\right)$$

where $h(\omega) = \omega - \nu_\omega (1 - \mathbb{E}[m(\omega)])$ and $\nu_\omega \geq 0$ is the Lagrange multiplier on the IC constraints that determine the choice of $\hat{d}$. This is the expression in proposition 5. □

Proof of proposition 7. First, I will show that, when ω-securities are available, the agent has a profitable deviation. Take the special case of section 1.4 and let ξ denote the payment schedule that solves (1.4). Consider the deviation where the agent sells some risk by buying a portofolio of ω-securities that pays off $-\kappa\omega/m(\omega)$, for a small $\kappa > 0$. From lemma 4, the price of this security is $-\kappa\mathbb{E}[\omega]/\mathbb{E}[m(\omega)] = 0$. If the agent buys this portfolio, the mean of x will be unaffected, but x becomes less correlated with ω. With a slight abuse of notation, let $F_{x|\omega}(x|\omega; k, k, -\kappa\omega/m(\omega), p(\cdot))$ be the conditional disitribution of x, when the portfolio $-\kappa\omega/m(\omega)$ is selected. Differentiating the agent's utility w.r.t. κ around $\kappa = 0$ (that is, around the point where the agent doesn't deviate) yields

$$\begin{aligned}
&\frac{\partial}{\partial\kappa}\left(\int u(\xi(x,\omega))\, dF_{x|\omega}\left(x|\omega; k, k, -\kappa\frac{\omega}{m(\omega)}, p(\cdot)\right) dF_\omega(\omega) - C(k)\right) \\
=\ & \int u(\xi(x,\omega))\left(\frac{\partial}{\partial\kappa} dF_{x|\omega}\left(x|\omega; k, k, -\kappa\frac{\omega}{m(\omega)}, p(\cdot)\right)\right) dF_\omega(\omega) \\
=\ & \int u(\xi(x,\omega))\left(-\frac{1}{\sigma_x}\frac{\omega x}{m(\omega)}\right) d\Phi(x)\, dF_\omega(\omega) > 0.
\end{aligned}$$

The last inequality comes from the optimal contract (1.4). Thus, $\bar{U}_\omega(k, p(\cdot)) \geq \int u(\xi(x,\omega))\, d\Phi(x)\, dF_\omega(\omega) - C(k)$ holds with a strict inequality.

I have to prove that ω-securities reduce the equilibrium level of k. A quick way to prove this result is to rewrite the contracting problem without using the transformation x, that is, I let the payment ξ be conditional on (Π, ω) instead of (x, ω). Let

$$L_{\Pi|\omega}\left(\Pi|\omega; \hat{k}, d = 0, p(\cdot)\right) = \frac{\partial F'_{\Pi|\omega}\left(\Pi|\omega; \hat{k}, d = 0, p(\cdot)\right)/\partial\hat{k}}{F_{\Pi|\omega}\left(\Pi|\omega; \hat{k}, d = 0, p(\cdot)\right)}$$

be the likelihood ratio of Π conditional on ω. Note that $L_{\Pi|\omega}$ depends only on the actual choice of the agent, $\hat{k}$, but not on the level suggested by the principal, k. Without restating the problem, from Holmstrom (1979), we know that the optimal payment will be such that the agent's utility is the average of $\tilde{u}\left(\left(\mu + \lambda L_{\Pi|\omega}\left(\Pi|\omega; \hat{k}, 0, p(\cdot)\right)\right)/m(\omega)\right)$. The reason why using (Π, ω) instead of

(x, ω) simplifies the proof is that now the outside option

$$\bar{U}_{\omega}(p(\cdot)) = \max_{\hat{k},\,\hat{d}} \int u(\xi(x,\omega))\, dF_{\Pi|\omega}\left(x|\omega; \hat{k}, \hat{d}, p(\cdot)\right) dF_{\omega}(\omega) - C\left(\hat{k}\right)$$

depends on the recommended k only through the contract ξ. In the contract of proposition 1.4, μ and λ are both increasing functions of k. Therefore, if a deviation is profitable for some k, that is, $\bar{U}_{\omega}(k, p(\cdot)) \geq \bar{u}$, then the same deviation must be profitable for a higher k. Formally, $\bar{U}_{\omega}(k', p(\cdot)) \geq \bar{U}_{\omega}(k, p(\cdot)) \geq \bar{u}$ for $k' \geq k$.

Finally, we have to prove that welfare decreases when ω-securities are available. The proof is analogous to the proof of proposition 3. The social planner solves the problem of choosing ξ, k, and c so as to maximize welfare in the economy. Formally, the social planner solves:

$$\max_{\xi,k,c} \int v(c(\omega))\, dF_{\omega}(\omega)$$

subject to:

$$\frac{\partial}{\partial \hat{k}} \int u(\xi(x,\omega))\, dF_{x|\omega}\left(x|\omega; k, \hat{k}, 0, p(\cdot)\right) dF_{\omega}(\omega)\bigg|_{\hat{k}=k} - C'(k) = 0,$$

$$\bar{U}_{\omega}(k, p(\cdot)) \leq \bar{u}$$

$$\int u(\xi(x,\omega))\, d\Phi(x)\, dF_{\omega}(\omega) - C(k) = \bar{u},$$

$$c(\omega) + \int \xi(x,\omega)\, d\Phi(x) = \int \left[x + \bar{R} + r\ k + (1-k)\,\omega\right] d\Phi(x),\ \forall \omega.$$

Compared to problem P(ω), the social planner has one extra control variable, $c(\omega)$, but he has to satisfy the resource constraint of the economy. Note that $c(\omega)$ appears only in the objective function and in the resource constraint. So, we can separate the problem by first choosing $c(\omega)$ to solve

$$\max_{c} \int v(c(\omega))\, dF_{\omega}(\omega)$$

subject to

$$c(\omega) + \int \xi(x,\omega)\, d\Phi(x) = \int \left[x + \bar{R} + r\ k + (1-k)\,\omega\right] d\Phi(x),\ \forall \omega.$$

Let $\varphi(\omega)$ be the Lagrange multiplier on the resource constraint. The first-order condition w.r.t. c is

$$m(\omega) \equiv v'(c(\omega)) = \varphi(\omega).$$

Now, conditional on $\varphi(\omega)$, we can solve the dual problem of choosing ξ and k to maximize the value of resources in the economy. This problem is the same as P(ω). Thus, the equilibrium of the economy is a solution to the planner's problem and $\bar{U}_{\omega}(k, p(\cdot)) \leq \bar{u}$ is a constraint on the planner's problem. Therefore, welfare is lower when ω-securities are available. $\square$

1.10.3 Both types of securities

Proof of Lemma 6. The proof of this result is similar to those of lemmas 1 and 4 combined with the analysis in Makowski (1979). $\square$

Proof of proposition 8. Assume that we are in the environment of section 1.5.1. The optimal contract is given in proposition 4. Assume now that the double-deviation is possible and its value is given by $\bar{U}_{\varepsilon\omega}(k, p(\cdot))$. Consider first a small increase of c_ε and, thus, p^ε (the case with c_ω is analogous). From proposition 4 we know that the optimal contract is increasing in the mean of x. This implies

$$\frac{\partial}{\partial p^\varepsilon}\bar{U}_{\varepsilon\omega}(k, p(\cdot)) \geq 0,$$

which tightens the constraint $\bar{U}_{\varepsilon\omega}(k, p(\cdot)) \leq \bar{u}$. Now, for higher values of k, the Lagrange multipliers μ and λ in 1.8 increase to satisfy the IC and IR constraint. From

$$\bar{U}_{\varepsilon\omega}(k, p(\cdot)) = \max_{\hat{k}, s^\omega} \int \tilde{u}\left(\frac{\mu + \lambda\ (r-\omega)\ x/\sigma_u}{m(\omega)}\right) dF_{x|\omega}\left(x|\omega; k, \hat{k}, s^\varepsilon = \varnothing, s^\omega \neq \varnothing, p(\cdot)\right) dF_\omega(\omega)$$

we see that if a deviation was profitable for a certain k, $\bar{U}_{\varepsilon\omega}(k, p(\cdot)) > \bar{u}$, then it will profitable also for $k + dk$. Formally,

$$\frac{\partial^2}{\partial k \partial p^\varepsilon}\bar{U}_{\varepsilon\omega}(k, p(\cdot)) \geq 0.$$

The latter proves that higher values of p^ε have a bigger effect on the cost of implementing a certain action when k is higher. In turn, this implies that the optimal level of k decreases with p^ε. $\square$

Proof of lemma 7. Consider a tax on the transactions in the securities market. A transaction in this context has to be interpreted as a trade between an issuer and a manager. In other words, I assume that issuers and managers will pay the tax any time they trade something, independently of the quantity of securities exchanged.

When transactions are not observable, the principal has to consider three possible deviations: not trading any security, trading both securities, and trading only ω-securities (double-deviation). Formally, let $\bar{U}_\varepsilon(k, p(\cdot))$, $\bar{U}_\omega(k, p(\cdot))$, and $\bar{U}_{\varepsilon\omega}(k, p(\cdot))$ denote the values of the three deviations, respectively. Also, let τ be the tax per transaction, then in equilibrium the fixed cost of transaction increases from c_ε to $c_\varepsilon + \tau$ (and from c_ω to $c_\omega + \tau$).

The social planner's problem is:

$$\max_\tau \max_{\xi, k} \int v'(c(\omega))\left(x + \bar{R} + r\ k + (1-k)\,\omega - \xi(x,\omega) + p^\varepsilon\right) d\Phi(x)\, dF_\omega(\omega)$$

subject to:

$$\frac{\partial}{\partial \hat{k}} \int u\left(\xi\left(x,\omega\right)\right) dF_{x|\omega}\left(x|\omega; k, \hat{k}, d^{\varepsilon} = -k\varepsilon, d_i = \varnothing\right) dF_{\omega}\left(\omega\right)\Big|_{\hat{k}=k} - C'\left(k\right) = 0,$$

$$\bar{U}_{\varepsilon}\left(k, p\left(\cdot\right)\right) \leq \bar{u},$$

$$\bar{U}_{\omega}\left(k, p\left(\cdot\right)\right) \leq \bar{u},$$

$$\bar{U}_{\varepsilon\omega}\left(k, p\left(\cdot\right)\right) \leq \bar{u},$$

$$\int u\left(\xi\left(x,\omega\right)\right) d\Phi\left(x\right) dF_{\omega}\left(\omega\right) - C\left(k\right) \geq \bar{u}.$$

Here, the tax τ affects prices both directly (the tax is imposed on each transaction) and through its effect on $c(\omega)$ and, thus, $m(\omega)$. However, note that in equilibrium the proceeds from tax are rebated to the representative investor. Thus, aggregate consumption $c(\omega)$ is not affected directly by τ, but only through the effect on $p(\cdot)$. Now, if we relax the problem by dropping the contraints on the three deviaitions, the problem is the same as P(ε). In section 1.5.1, I proved that the optimal contract of this proble is such that the agent wants to buy full insurance. Thus, at $\tau = 0$, we have that $\bar{U}_{\varepsilon}(k, p(\cdot)) = \bar{u}$. However, when $\tau = 0$, ω-securities represent a profitable deviation for the agent, that is, $\bar{U}_{\omega}(k, p(\cdot)) < \bar{u}$. This implies that a small positive $\tau > 0$ has a second order effect on $\bar{U}_{\varepsilon}(k, p(\cdot))$ but a first order effect on $\bar{U}_{\omega}(k, p(\cdot))$. Thus, a small positive $\tau > 0$ increase welfare in the economy. □

Proof of proposition 9. The proof of this result follows from the intuition given in the text. When transactions are observable, then the principal can limit the manager to make only one transactions by punishing him (by setting $\xi_i = -\infty$) for deviating. Thus, the manger will always trade one and only one security. Now, except for choosing a different investment fraction k, the only other possible deviation is the double-deviation of trading only ω-securities. Thus, the only constraint on the optimal contracting problem is $\bar{U}_{\varepsilon\omega}(k, p(\cdot)) \leq \bar{u}$. Also, note that the value of τ doesn't directly affect $\bar{U}_{\varepsilon\omega}(k, p(\cdot))$ since the agent is trading only one security. In turn, this implies that this problem is a relaxed version of the problem of lemma 7 and the result follows. □

Chapter 2

Cycles, Gaps, and the Social Value of Information [1]

2.1 Introduction

Market participants pay close attention to public signals regarding the state of the economy, like those embodied in central-bank communications, macroeconomic statistics, surveys of consumer confidence, or news in the media. Given how noisy these signals can be, the market's heightened response to them often feels "excessive". In an influential *AER* article, Morris and Shin (2002) have argued that this is due to the sunspot-like role that public news play in environments with dispersed information: by helping agents coordinate their choices, noisy public signals can have a destabilizing effect on the economy, contributing to higher volatility and lower welfare.

Motivated by these observations, this paper seeks to understand the welfare effects of information within the context of business cycles. To this goal, we abandon the "beauty-contest" framework used by Morris and Shin (2002) and much of the subsequent literature.[2] Although the findings of this work are customarily interpreted in a macroeconomic context, the lack of micro-foundations in this prior work renders such interpretations largely premature. By contrast, we employ a micro-founded framework that nests the neoclassical backbone of modern DSGE models. In so doing, we develop a clean theoretical benchmark for the welfare effects of information within the context of business cycles—one that provides a novel, and sharp, answer to the question of interest.

Framework. We follow the New-Keynesian paradigm in allowing for product differentiation and monopolistic competition. We nevertheless abstract from nominal rigidities, thereby focusing, a fortiori, on flexible-price allocations. We do so because this is always an excellent benchmark for normative questions: the welfare properties of sticky-price allocations hinge on the welfare properties of the underlying flexible-price allocations. Finally, we allow for multiple type of shocks, including

[1]The results in this chapter are joint work with George Marios Angeletos and Jennifer La'O

[2]This includes three sequels in *AER*: a critique by Svensson (2005), a response by Morris et al. (2005), and a recent article by James and Lawler (2011).

shocks to technologies and preferences, as well as shocks to monopoly markups and labor wedges.

This last modeling choice brings our analysis close to the pertinent DSGE paradigm (Christiano, Eichenbaum, and Evans, 2005, Smets and Wouters, 2007). Most importantly, it helps accommodate two distinct notions of fluctuations. Technology and preference shocks capture fluctuations that involve no variation in the "output gap" between flexible-price and first-best allocations. By contrast, shocks to markups and labor wedges capture fluctuations that are tied to market distortions and that manifest with variation in the aforementioned "output gap". This distinction is known to be a key determinant of both the desirability of output-stabilization policies and the welfare costs of the business cycle: see, inter alia, Goodfriend and King (1997), Rotemberg and Woodford (1997), Adao, Correia and Teles (2003), Khan, King and Wolman (2003), Benigno and Woodford (2005), and Gali, Gertler, and Lopez-Salido (2007). As we show in this paper, this distinction is also central to understanding the welfare effects of information.

Turning to the information structure, we let agents observe, not only noisy signals of the exogenous shocks, but also noisy indicators of the endogenous economic activity. This is key to both the robustness and the applicability of our insights. Macroeconomic statistics and financial prices are noisy indicators of the choices and opinions of other agents. The information that these signals contain regarding the underlying structural shocks is thus endogenous to equilibrium behavior. This gives rise to informational externalities which, in general, can have important implications for the normative properties of the economy (Amador and Weill, 2010, 2011; Vives, 2008, 2011). Our analysis takes care of these complications and permits us to interpret public information as a signal of either the exogenous shocks or, more realistically, of the endogenous state of the economy.

Results. Our first result decomposes equilibrium welfare in terms of discrepancies, or "gaps", between equilibrium and first-best allocations. This decomposition, which extends related results from complete-information models (e.g., Woodford, 2003b, Gali, 2008), is instrumental in characterizing the welfare effects of either the underlying shocks or the available information. Under complete information, the aforementioned discrepancies obtain only because of product and labor-market distortions. With incomplete information, they obtain also because of noise in the available information. Either way, the consequent welfare losses manifest at the macro level as volatility in the aggregate output gap, and in the cross section as excess dispersion in relative prices.

Consider now the case where the business cycle is driven by technology and preference shocks. If information were complete, these shocks would cause efficient fluctuations: both the volatility of aggregate output gaps and the excess dispersion in relative prices would have been zero. It follows that, when information is incomplete, the welfare effects of information hinge entirely on the impact of noise: any correlated noise contributes to volatility in aggregate output gaps, while any idiosyncratic noise contributes to excess dispersion in relative prices.

As more precise public information motivates agents to pay less attention to their private information, the dispersion in relative prices falls, contributing to higher welfare. However, as agents pay more attention to public information, aggregate output gaps may become more volatile, contributing to lower welfare. The overall effect thus looks ambiguous at first glance. Nonetheless, we show that the beneficial effect on price dispersion necessarily outweighs any potentially adverse effect on aggregate volatility, guaranteeing that welfare necessarily improves with more information.

This result holds true despite the coordinating role of public information. The aggregate demand externalities that are embedded in our business-cycle framework induce a form of strategic complementarity that is akin to the one in the "beauty contests" studied by Morris and Shin (2002) and subsequent work. Nonetheless, the welfare lessons of this prior work are inapplicable: public information is found to be welfare improving no matter the degree of strategic complementarity.

Two observations are key to understanding these findings. First, the negative welfare effect of public information within "beauty contests" a la Morris and Shin (2002) hinges entirely on the presumption that its coordinating role is socially undesirable. By contrast, the coordinating motives that originate from aggregate demand externalities and that are thus embedded in conventional DSGE models like ours are fully warranted from a social perspective. Second, as long as the underlying market distortions are state-invariant, these distortions do not interfere with the response of the economy to any noisy information about the underlying preference and technology shocks. The first observation explains why the insights of Morris and Shin (2002) are inapplicable; the second explains why information about technology and preference shocks is necessarily welfare improving.

With these insights in mind, we next study the social value of information when the business cycle is driven by shocks to monopoly power and labor-market wedges—that is, by shocks that interfere with the level of market inefficiency. For this case, we show that more noise *reduces* the welfare losses associated with the volatility of aggregate output gaps and the excess dispersion in relative prices. In this sense, information is detrimental for welfare.

At the same time, information has a countervailing effect though the mean level of economic activity: by reducing the uncertainty faced in equilibrium, more precise information brings the mean value of aggregate output closer to the optimal one, contributing to higher welfare. The strength of this countervailing effect, however, depends on the severity of the market distortion: the smaller the level of the distortion, the smaller the benefits of any given reduction in it. We thus show that, for the case of inefficient fluctuations, the overall effect of public information on welfare is negative as long as the mean level of monopoly or other market distortions is not too large.

A simple punchline thus emerges: the welfare effects of information hinge essentially on the same conditions as the optimality of flexible-price allocations. Our results thus provide a direct mapping from the view one may hold regarding the need for output stabilization to the inference one should make regarding the welfare effects of information regarding the state of the economy, whether this information is provided by policy makers, statistical agencies, the media, or the markets.

A numerical exercise is used to shed further light. In line with our theoretical results, the *sign* of the welfare effects of information is pinned down by the relative contribution of technology and markup shocks. Nonetheless, aggregate demand externalities—and the strategic complementarity obtains from them—emerge as a key determinant of the *magnitude* of these effects. Finally, when the contribution of the various shocks in our model is matched to the one estimated by Smets and Wouters (2007) for the US economy, technology shocks are sufficiently prevalent that the social value of information is positive: more information improves welfare.

Related literature. Our paper adds to a large literature that followed the influential contribution of Morris and Shin (2002). Much of this work—including Svensson (2005), Angeletos and Pavan (2007, 2009), Morris and Shin (2007), Cornand and Heinemann (2008), Myatt and Wallace (2009), and James and Lawler (2011)—continues to employ the same abstract game as Morris and Shin, or certain variants of it. By contrast, our paper studies the welfare effects of information within the class of micro-founded DSGE economies that rest at the core of modern macroeconomic theory. The discipline imposed by the micro-foundations of this particular class of economies explains the sharp contrast between our results and those of the aforementioned work.

The framework we use is borrowed from Angeletos and La'O (2009); similar micro-foundations underlie, inter alia, Woodford (2003a) and Lorenzoni (2010). This prior work shows how dispersed information can have profound implications on the positive properties of the business cycle. Here, we shift focus to the normative question of how information impacts welfare. We thus complement Hellwig (2005) and Roca (2010), which are concerned with the same question but focus on monetary shocks—shocks that matter only when prices are sticky and monetary policy fails to offset them. Related are also Amador and Weill (2010, 2011) on the interaction between public information and social learning, as well as Angeletos and La'O (2011) and Wiederholt and Paciello (2011) on optimal monetary policy with informational frictions. We elaborate on these relations in Sections 7 and 9.

Finally, the methodological approach we take in this paper builds on Angeletos and Pavan (2007). That paper was the first to highlight that, in general, the welfare effects of information hinge on the relation between equilibrium and efficient allocations, and to indicate the potential importance of different types of shocks. Nevertheless, the analysis of that paper was confined within an abstract class of linear-quadratic games. By contrast, the contribution of the present paper is squarely on the applied front. To the best of our knowledge, our paper is the first to study the welfare effects of information along the flexible-price allocations of a canonical, micro-founded DSGE model.

Layout. The rest of the paper is organized as follows. Section 2 introduces our framework. Section 3 characterizes the equilibrium and Section 4 decomposes welfare. Sections 5 and 6 study the welfare effects of information for the cases of, respectively, technology/preference shocks and markup/labor-wedge shocks. Section 7 extends the analysis from signals of the exogenous shocks to signals of the endogenous state of the economy. Section 8 conducts a suggestive numerical exercise. Section 9 concludes with a translation of our results to sticky-price models, and with a discussion of directions for future research.

2.2 The model

Our framework builds on Angeletos and La'O (2009), adding incomplete information to the flexible-price allocations of an elementary DSGE model. As in Lucas (1972), there is a continuum of "islands", which define the "geography" of information: information is symmetric within islands, but asymmetric across islands. Each island is inhabited by a continuum of firms, which specialize in the production of differentiated commodities. Finally, there is a representative household, or "family", consisting of a consumer and a continuum of workers, one worker per island. Islands are indexed by $i \in I = [0, 1]$; firms and commodities by $(i, j) \in I \times J$; and periods by $t \in \{0, 1, 2, ...\}$.

Each period has two stages. In stage 1, workers and firms decide how much labor to, respectively, supply and demand in their local labor market, and local wages adjust so as to clear any excess demand. At this point, workers and firms know their local fundamentals, but imperfect information regarding the fundamentals and the level of economic activity in other islands. After employment and production choices are sunk, workers return home and the economy transitions to stage 2. At this point, all information that was previously dispersed becomes publicly known, and centralized markets operate for the consumption goods. This two-stage structure permits us to introduce dispersed information while maintaining the convenience of a representative consumer.

Households. The utility of the representative household is given by

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left[U(C_t) - \int_I S^n_{i,t} V(n_{i,t}) di, \right]$$

where

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \qquad \text{and} \qquad V(n) = \frac{n^{1+\epsilon}}{1+\epsilon}.$$

with $\gamma, \epsilon \geq 0$. Here, $n_{i,t}$ is labor effort on island i during stage 1 of period t, $S^n_{i,t}$ is an island-specific shock to the disutility of labor, and C_t is aggregate consumption. The latter, which also defines the numeraire used for wages and commodity prices, is given by the following nested CES structure:

$$C_t = \left[\int_I S^c_{i,t} (c_{i,t})^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}}$$

where $S^c_{i,t}$ is an island-specific shock to the utility of the goods produced by island i and $c_{i,t}$ is a composite of the goods produced by island i, defined by

$$c_{i,t} = \left[\int_J (c_{ij,t})^{\frac{\eta_{it}-1}{\eta_{it}}} dj \right]^{\frac{1}{\eta_{it}}},$$

with $c_{ij,t}$ denoting the quantity consumed in period t of the commodity j from island i. Here, ρ identifies the elasticity of substitution across the consumption composites of different islands, while η_{it} identifies the elasticity of substitution across the goods produced within a given island i.

As will become clear shortly, ρ controls the strength of aggregate demand externalities (the sensitivity of optimal firm profits to aggregate output), while η controls the degree of monopoly power. The pertinent literature often imposes (implicitly) that $\rho = \eta$, thereby confounding the notion of demand externalities with the notion of monopoly power. By contrast, we let $\rho \neq \eta$ so that we can separate these two distinct notions. Finally, by letting η be random and abstracting from any heterogeneity within islands, we introduce a pure form of markup shocks that cause variation in equilibrium allocations without affecting first-best allocations.

Households are diversified in the sense that they own equal shares of all firms in the economy. The budget constraint of household h is thus given by the following:

$$\int_I \int_J p_{ij,t} c_{ij,t} dj dj + B_{t+1} \leq \int_I \int_J \pi_{ij,t} di dj + \int_I (1 - \tau_{i,t}) w_{it} n_{i,t} di + R_t B_t + T_t,$$

Here, $p_{ij,t}$ is the period-t price of the commodity produced by firm j on island i, $\pi_{ij,t}$ is the period-t profit of that firm, w_{it} is the period-t wage on island i, R_t is the period-t nominal gross rate of return on the riskless bond, and $B_{h,t}$ is the amount of bonds held in period t.

The variables $\tau_{i,t}$ and T_t are exogenous to the representative household and the firms, but satisfy the following restriction at the aggregate:

$$T_t = \int_I \tau_{it} w_{it} n_{it} di.$$

One can thus readily interpret τ_{it} as an island-specific tax on labor income and T_t as the resulting aggregate tax revenues that are distributed back to households in lump-sum transfers. Alternatively, we can consider a variant of our model with monopolistic labor markets as in Blanchard and Kiyotaki (1987), in which case τ_{it} could re-emerge as an island-specific markup between the wage and the marginal revenue product of labor. In line with much of the DSGE literature, we interpret $1 - \tau_{it}$ more generally as a "labor wedge" or a "labor-market distortion".

The objective of each household is simply to maximize expected utility subject to the budget and informational constraints faced by its members. Here, one should think of the worker-members of each family as solving a team problem: they share the same objective (family utility) but have different information sets when making their labor-supply choices. Formally, at the beginning of stage 1 the household sends off its workers to different islands with bidding instructions on how to supply labor as a function of (i) the information that will be available to them at that stage and (ii) the wage that will prevail in their local labor market. In stage 2, the consumer-member collects all the income that the worker-members have earned and decides how much to consume in each of the commodities and how much to save (or borrow) in the riskless bond.

Firms. The output of firm j on island i during period t is given by

$$y_{ij,t} = A_{i,t} n_{ij,t}$$

where $A_{i,t}$ is the productivity in island i and $n_{ij,t}$ is the firm's employment.[3] The firm's realized profit is given by $\pi_{ij,t} = p_{ij,t}y_{ij,t} - w_{i,t}n_{ij,t}$. Finally, the objective of the firm is to maximize its expectation of $U'(C_t)\pi_{ij,t}$, the representative consumer's valuation of its profit.

Markets. Labor markets operate in stage 1, while product markets operate in stage 2. The corresponding clearing conditions are as follows:

$$\int_J n_{ij,t}dj = n_{i,t}\ \forall i \qquad \text{and} \qquad c_{ij,t} = y_{ij,t}\ \forall (i,j)$$

Asset markets also operate in stage 2, when all information is commonly shared. This guarantees that asset prices do not convey any information. The sole role of the bond market in the model is then to price the risk-free rate. Moreover, because our economy admits a representative consumer, allowing households to trade risky assets in stage 2 would not affect any of the results.

Shocks and information. Each island is subject to multiple shocks: technology shocks are captured by A_{it}, preference shocks by S_{it}^n and S_{it}^c, markup shocks by η_{it}, and labor-wedge shocks by τ_{it}. These shocks have both aggregate and idiosyncratic components.

Turning to the information structure, we note that, once all agents meet in the centralized commodity market that takes place in stage 2, they can aggregate their previously dispersed information and can therefore reach common knowledge about the aforementioned shocks. Nonetheless, such common knowledge may be missing in the beginning of the period, when firms and workers have to make their employment and production choices.

Our specification of the information structure is otherwise flexible enough to allow for multiple private and public signals, some of which could be the product of noisy indicators of aggregate economic activity. We nevertheless restrict the information structure to be Gaussian in order to obtain a closed-form characterization of equilibrium allocations and welfare. Without this restriction, our results can be re-interpreted as local approximations, pretty much as in Woodford (2003) and most other log-linearized DSGE models. We spell out the details of this Gaussian specification in due course, making clear the particular results that depend on it.

Aggregates and equilibrium. We henceforth normalize nominal prices so that the "ideal" price index is constant:

$$P_t \equiv \left[\int p_{it}^{1-\rho}di\right]^{\frac{1}{1-\rho}} = 1.$$

[3] The assumption of linear returns to labor is consistent with the idea that, at business-cycle frequencies, the variation in the stock of capital is negligible and that the rate of capital utilization is proportional to hours. In any event, this assumption is only for expositional simplicity: the results extend directly to decreasing returns. In fact, they also extend to increasing returns, provided that the curvature that is introduced in the typical firm's problem by the downward slopping demands is enough to preserve the convexity of that problem and, in so doing, to also preserve the uniqueness of the equilibrium.

We also define aggregate output Y_t and employment N_t as follows:

$$Y_t \equiv \left\{ \int S_{it}^c y_{it}^{\frac{\rho-1}{\rho}} di \right\}^{\frac{\rho}{\rho-1}} \quad \text{and} \quad N_t \equiv \int n_{it} di.$$

Our definition of aggregate output is thus consistent with the usual reinterpretation of the Dixit-Stiglitz framework that lets the differentiated commodities be intermediate inputs in the production of single final consumption good. Finally, an equilibrium is defined as a collection of wages, commodity prices, and employment, production, and consumption plans such that (i) wages clear the local labor markets that operate in each island during stage 1; (ii) commodity prices clear the centralized product markets that operate during stage 2; (iii) employment and production levels are optimally chosen conditional on the information that is available in stage 1; and (iv) consumption levels are optimally chosen conditional on the information that is available on stage 2.

2.3 Equilibrium and first-best allocations

To characterize the equilibrium, consider the behavior of firms and workers in a given island i and a given period t.[4] When firms decide how much labor to employ and how much to produce during stage 1, they understand that they are going to face a downward-slopping demand in stage 2. They therefore seek to to equate the local wage with the expected marginal revenue product of their labor, targeting an optimal markup over marginal cost. Workers, on their part, equate their disutility of effort with the expected marginal value of the extra income they can provide their family. It follows that the period-t equilibrium production levels of any given island i are pinned down by the following condition:

$$S_{it}^n V' \left(\frac{y_{it}}{A_{it}} \right) = (1 - \tau_{i,t}) \left(\frac{\eta_{it} - 1}{\eta_{it}} \right) \mathbb{E}_{it} \left[S_{it}^c U'(Y_t) \left(\frac{y_{it}}{Y_t} \right)^{-\frac{1}{\rho}} \right] A_{it}, \tag{2.1}$$

To interpret this condition, note that the LHS is the marginal disutility of effort, while the RHS is the product of the labor wedge, times the reciprocal of the monopoly markup, times the expected marginal value of the marginal product of labor. This condition therefore equates private costs and benefits. Finally, note that expectations are taken over Y_t: firms and households alike are uncertain about the ongoing aggregate economic activity because, and only because, information is dispersed.

For comparison, the first-best allocation satisfies the following condition:

$$S_{it}^n V' \left(\frac{y_{it}}{A_{it}} \right) = S_{it}^c U'(Y_t) \left(\frac{y_{it}}{Y_t} \right)^{-\frac{1}{\rho}} A_{it}. \tag{2.2}$$

It follows that equilibrium allocations can deviate from first-best allocations, not only because of monopoly power and labor-market distortions, but also because of errors in expectations of aggregate

[4]The characterization of the equilibrium follows from Angeletos and La'O (2009). The contribution of the present paper rests in the welfare analysis of the subsequent sections.

output—and hence because of the noise in the available information. Our subsequent analysis will therefore explore how the resulting welfare losses vary with the precision of the available information.

Before proceeding to this, however, it is worth highlighting a certain isomorphism between our business-cycle framework and the class of coordination games studied by Morris and Shin (2002) and others. Assuming a Gaussian information structure, we can restate condition (2.1) as follows:

$$\log y_{it} = const + (1-\alpha)\, f_{it} + \alpha\, \mathbb{E}_{it}\left[\log Y_t\right] \tag{2.3}$$

where $f_{it} \equiv \frac{1}{\epsilon+\gamma} \log \left[A_{it}^{1+\epsilon} \frac{S_{it}^c}{S_{it}^n} (1-\tau_{i,t}) \left(\frac{\eta_{it}-1}{\eta_{it}}\right) \right]$ is a composite of the underlying shocks and

$$\alpha \equiv \frac{\frac{1}{\rho} - \gamma}{\frac{1}{\rho} + \epsilon} < 1. \tag{2.4}$$

The general equilibrium of our economy can therefore be interpreted as the PBE of a "beauty contest" among the different islands of the economy, with an island's best response given by condition (2.3) and the corresponding degree of strategic complementarity given by α.

At first glance, this isomorphism might suggest that the welfare lessons of Morris and Shin (2002) apply to our framework and more generally to conventional business-cycle models; we explain why this is not the case in the following sections. At the same time, this isomorphism captures an important truth: the incentives of the typical economic agent—whether a firm, a worker, or a consumer—crucially depend on expectations of the aggregate choices of all other agents.

This kind of interdependence—a form of strategic complementarity—is embedded in any modern DSGE model and reflects the combination of at least two kinds of general-equilibrium effects. On the one hand, an increase in macroeconomic activity raises the demand faced by each firm, which other things equal stimulates firm profits, production, and employment. On the other hand, an increase in income discourages labor supply and raises real wages which, other things equal, has the opposite effect on firm profits, production, and employment. The former effect formalizes the familiar Keynesian notion of aggregate demand externalities; the latter captures the reaction of wages on the labor-supply side. The overall feedback is thus positive ($\alpha > 0$) if and only if the demand-side effect dominates—a property that seems likely to hold in practice and rests at the heart of Keynesian thinking. Although not required for our key results,[5] we henceforth impose this restriction in order to simplify the exposition.

Assumption. *The equilibrium features strategic complementarity in the sense that local output increases with beliefs of aggregate output (that is, $\alpha > 0$).*

[5]As shown in the Appendix, our decomposition of welfare in the next section (Proposition 1) and our key welfare lessons (Theorems 1 and 2) hold true even if $\alpha < 0$.

2.4 Welfare

We now move to the core of our contribution, starting with a decomposition of the channels through which the various structural shocks, and the available information about them, affect welfare.

Take any feasible allocation and let y_{it} and Y_t denote the associated local and aggregate levels of output. As standard, welfare is defined by the ex-ante utility of the representative household. Since $C_t = Y_t$ and $n_{it} = y_{it}/A_{it}$, this implies that welfare is given by the following:

$$\mathcal{W} \equiv \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left\{U(Y_t) - \int S_{it} V\left(\frac{y_{it}}{A_{it}}\right) di\right\}\right]$$

Next, let y_{it}^* and Y_t^* denote the (full-information) first-best levels of, respectively, local and aggregate output. Define then the local and aggregate output gaps by the following:

$$\log \tilde{y}_{it} \equiv \log y_{it} - \log y_{it}^* \qquad \text{and} \qquad \log \tilde{Y}_t \equiv \log Y_t - \log Y_t^*$$

Finally, let $\theta_{it} \equiv \log\left(A_{it}, S_{it}^c, S_{it}^n\right)$, and let Θ_t denote the cross-sectional average of θ_{it}. We impose the following joint restriction on these shocks and the allocation under consideration.

Assumption. *The aggregate variables $(\Theta_t, \log Y_t)$ and the local variables $(\theta_{it}, \log y_{it})$ are jointly Normal, and the latter are i.i.d. across i conditional on the former.*

As will become clear shortly, this joint log-normality restriction is automatically satisfied by the equilibrium allocation as long as the underlying information structure is Gaussian. The next result, however, provides us with a convenient characterization of welfare that holds true irrespectively of whether the allocation under consideration is an equilibrium one or not and that obtains in any given allocation. This characterization is exact as long as the aforementioned assumption holds, but can also be interpreted more generally as a second-order approximation.

Proposition 11. *Welfare is given by*

$$\mathcal{W} = \mathbb{E}\left[\sum_t \beta^t W_t^* \Delta_t \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda_t\right\}\right]$$

where W_t^* is the period-t utility that obtains at the first best, Δ_t captures the first-order welfare losses that obtain from suboptimality in the mean level of aggregate output, and Λ_t captures the second-order welfare losses that obtain from uncertainty.[6]

For the first-order losses, we have that

$$\Delta_t = \Delta(\delta_t) \equiv \frac{U(\delta_t) - V(\delta_t)}{U(1) - V(1)}$$

[6]By uncertainty here we mean both the one associated with the underlying structural shocks and the one associated with the noise in the available information.

where δ_t is the ratio between $\mathbb{E}[Y_t|\Theta_{t-1}]$, the mean (predictable) level of aggregate output that obtains in the allocation under consideration, and the one that would have maximized welfare.

For the second-order losses, we have that

$$\Lambda_t = \Lambda(\Sigma_t, \sigma_t) \equiv \Sigma_t + \frac{1}{1-\alpha}\sigma_t$$

where $\alpha < 1$ is defined as in (2.4) and where

$$\Sigma_t \equiv Var[\tilde{Y}_t|\Theta_{t-1}] \qquad \text{and} \qquad \sigma_t \equiv Var[\tilde{y}_{it}|\Theta_t]$$

measure, respectively, the volatility of aggregate output gaps and the cross-sectional dispersion of local output gaps (or, equivalently, the excessive dispersion in relative prices).

This proposition generalizes related results that have been established in canonical, complete-information, macroeconomic models (e.g., Woodford, 2003b, Gali, 2008). The only key difference is that welfare losses relative to the first best may now originate, not only in markup and labor-wedge shocks, but also in noisy information. This difference, however, does not interfere with the insight that *any* type of welfare loss can conveniently be represented in terms of the associated gaps between equilibrium and first-best allocations. Establishing the robustness of this simple, but important, insight to imperfect information is the first key result of our paper.

To elaborate on the origins of these welfare losses, let us henceforth focus on equilibrium allocations and, to start, suppose that there are no shocks at either the macro or the micro level. In this case, there is no variation in output gaps at either the aggregate or the cross section ($\Sigma_t = \sigma_t = 0$), guaranteeing that all second-order welfare losses vanish ($\Lambda_t = 0$). Yet, first-order losses obtain as long as there is a positive markup or a positive labor wedge (in which case $\delta_t < 1$ and $W_t^* \Delta_t < W_t^*$).[7]

Next, consider the welfare implications of uncertainty. To start, suppose that information about the underlying shocks is perfect—the conventional scenario in the pertinent literature. In this case, the response of equilibrium allocations to productivity and taste shocks is efficient. This is the familiar result that flexible-price allocations are efficient in standard New-Keynesian models when the business cycle is driven only by productivity or taste shocks. It follows that the gap between equilibrium and first-best allocations can vary, whether in the aggregate or in the cross-section, only when there are shocks to monopoly markups and/or to labor wedges. Any *aggregate* variation in monopoly markups and labor wedges is then manifested in Σ_t, the volatility of aggregate output gaps, while any *idiosyncratic* variation in such markups and wedges is manifested in σ_t, the inefficient component of the dispersion in relative prices.

Finally, consider the case where information about the underlying shocks is imperfect. In this case, Σ_t and σ_t continue to capture the volatility and dispersion effects of any markup and labor-

[7]When $\gamma < 1$, W_t^* is positive and Δ_t is strictly concave in δ_t. When instead $\gamma > 1$, W_t^* is negative and Δ_t is strictly convex. Either way, however, the product $W_t^* \Delta_t$ is strictly concave and reaches its unique maximum at $\delta = 1$.

wedge shocks, to the extent that these shocks are present and the economy reacts to them. However, this effect now interacts with the incompleteness of information about these shocks. For example, less information has the potential of *reducing* both Σ_t and σ_t to the extent that it dampens the response of equilibrium allocations to the underlying markup and labor-wedge shocks. Intuitively, if agents have little information about this kind of shocks, they will "fail" to react to them, which is undesirable from a private perspective but desirable from a social perspective.

At the same time, less information has the potential of *raising* Σ_t to the extent that it dampens the response of equilibrium allocations to the underlying productivity and taste shocks. Intuitively, if agents have little information about this kind of shocks, they will again "fail" to react to them–but now this failure increases the gap between equilibrium and first best and hence is socially undesirable. Finally, any noise in information by itself can contribute to variation in either aggregate or local output gaps, whether this information regards markup and labor wedge shocks, or productivity and taste shocks. Noise by itself can thus play a role akin to markup or labor-wedge shocks.

Furthermore, how the aforementioned effects will get manifested in Σ_t and σ_t is bound to depend on whether information is public or private—or, equivalently, on whether the associated noise is correlated or idiosyncratic. For example, an increase in the precision of the public information is likely to reduce the noise-driven dispersion in relative prices by motivating people to shift their attention away from private sources of information. At the same time, the volatility of aggregate output gaps may actually increase as people pay more attention to noisy public news—and perhaps the more so if, because of the underlying aggregate demand externalities, such public signals end up playing a coordinating role akin to sunspots.

This discussion suggests that, in general, the effects of information on volatility, dispersion, and welfare are likely to hinge on (i) the nature of the underlying shocks, (ii) the extent to which information is public or private, and (iii) the strength of aggregate demand externalities, as summarized in α. We explore, and qualify, these intuitions in the subsequent sections.

2.5 Shocks to technologies and preferences

In this section we study the welfare effects of information for the special case in which the business cycle is driven only by preference and technology shocks. In particular, we impose that $\eta_{it} = \bar{\eta}$ and $\tau_{it} = \bar{\tau}$ for all islands, dates, and states, thereby ruling variation in either the monopolistic markup or the labor wedge. This helps us capture more generally the scenario in which the business cycle would have been efficient—in the sense that the output gap would be constant—had information been complete. It is this scenario that rests at the heart of the "divine coincidence" in modern New-Keynesian models; and it is this scenario that we focus on in this section.

To obtain closed-form solutions, we henceforth impose a Gaussian information structure. We also start with the case where agents observe only signals of the exogenous shocks. Finally, to simplify the exposition, we focus on technology shocks; the analysis for preferences shocks is identical modulo a change in notation/interpretation.

The log of local productivity, $a_{it} \equiv \log A_{it}$, is given by

$$a_{it} = \bar{a}_t + \xi_{it},$$

where $\bar{a}_t \equiv \log \bar{A}_t$ is the aggregate productivity shock and ξ_{it} is an idiosyncratic productivity shock. The latter is Normally distributed with unconditional mean 0 and variance $\sigma_\xi^2 \equiv 1/\kappa_\xi$, orthogonal to $\bar{a}_t$, and i.i.d. across islands and time. Aggregate productivity, on the other hand, is given by

$$\bar{a}_t = \chi_t + \nu_t,$$

where $\chi_t = \chi(\bar{a}_{t-1}, \bar{a}_{t-2}, ...) \equiv \mathbb{E}[\bar{a}_t|\bar{a}_{t-1}, \bar{a}_{t-2}, ...]$ is the component of the current productivity shocks that is predictable on the basis of past productivity shocks, and ν_t is a Normal innovation, i.i.d. over time, independent of any other shock, and with mean 0 and variance $\sigma_a^2 \equiv 1/\kappa_a$.

Note that local productivity a_{it} is itself a private signal of aggregate productivity $\bar{a}_t$. Each agent (or island) may have other sources of private information about the underlying productivity shock. We summarize all the private (local) information of island i regarding the current aggregate productivity shock $\bar{a}_t$ in a sufficient statistic x_{it} such that

$$x_{it} = \bar{a}_t + u_{it},$$

where u_{it} is idiosyncratic noise, Normally distributed, orthogonal to $\bar{a}_t$ and i.i.d. across i and t, with mean 0 and variance $\sigma_x^2 \equiv 1/\kappa_x$. In addition to this private information, every agent has also access to public information about the aggregate shock $\bar{a}_t$. This public information is summarized in a statistic z_t such that

$$z_t = \bar{a}_t + \varepsilon_t, \tag{2.5}$$

where ε_t is noise, Normally distributed with mean 0 and variance $\sigma_z^2 \equiv 1/\kappa_z$, and orthogonal to all other variables. The scalars κ_x and κ_z parameterize the precisions of available private and public information regarding the underlying productivity shock; the question of interest therefore reduces to the comparative statics of equilibrium welfare with respect to κ_z.

The above log-normal structure permit us to obtain a closed-form solution to both first-best and equilibrium allocations. In particular, it is easy to check that the first best satisfies[8]

$$\log y_{it}^* = \Psi \bar{a}_t + \psi(a_{it} - \bar{a}_t) \qquad \text{and} \qquad \log Y_t^* = \Psi \bar{a}_t \tag{2.6}$$

where $\Psi \equiv \frac{1+\epsilon}{\epsilon+\gamma}$ measures the response of equilibrium output to aggregate productivity shocks, while $\psi \equiv (1-\alpha)\Psi \equiv \frac{1+\epsilon}{\epsilon+1/\rho}$ measures the response to idiosyncratic shocks. The equilibrium, on the other

[8]The formulas in conditions (2.6) through (2.8) are correct up to constants that are spelled out in the Appendix but are omitted in the main text for expositional simplicity.

hand, satisfies

$$\log y_{it} = \varphi_a a_{it} + \varphi_x x_{it} + \varphi_z z_t + \varphi_{-1}\bar{a}_{t-1}, \tag{2.7}$$

where $\varphi_a, \varphi_x, \varphi_z$, and φ_{-1} are positive scalars that are determined by α, Ψ, and the information structure (see the Appendix for a detailed proof and the characterization of these coefficients). The corresponding aggregate level of output is given by

$$\log Y_t = \Phi\bar{a}_t + \varphi_z \varepsilon_t + \varphi_{-1}\bar{a}_{t-1}, \tag{2.8}$$

where $\Phi \equiv \varphi_a + \varphi_x + \varphi_z$ is positive but lower than Ψ.

The fact that $\Phi < \Psi$ means that incomplete information dampens the response of the economy to the underlying aggregate productivity (or taste) shocks. This property is akin to how incomplete information dampens the response of nominal prices to monetary shocks in Woodford (2003), Nimark (2008), and Lorenzoni (2010). But whereas the results of those papers rest entirely on the failure of monetary policy to replicate flexible-price allocations, the fact we document here underscores how incomplete information may distort the response of the economy to technology and preference shocks even when prices are flexible—or, equivalently, when prices are sticky but monetary policy succeeds in replicating flexible-price allocations, which is actually the optimal thing to do as long as the business cycle is driven only by these kind of shocks.[9]

This subdued response of equilibrium allocations to the underlying productivity shocks means that the aggregate output gap is now variable and negatively correlated with these shocks, or equivalently positively correlated with the business cycle: the booms and recessions caused by these shocks are too shallow relative to the first best. At the same time, the noise in the public signal z_t (or more generally any correlated errors in people's beliefs about aggregate economic activity) add independent variation in output gaps: a fraction of the booms and recessions is now driven by pure noise. Both of these effects contribute towards volatility in aggregate output gaps. Finally, the idiosyncratic noise in the private signals x_{it} (or more generally any idiosyncratic errors in the aforementioned beliefs) causes variation in local output gaps in the cross-section of the economy, contributing to inefficent dispersion in relative prices. The welfare effects of public information therefore hinge on the comparative statics of Σ_t and σ_t, which we study next.

Proposition 12. *(i) An increase in the precision of public information reduces relative-price dispersion and has a non-monotone effect on the volatility of aggregate output gaps:*

$$\frac{\partial \sigma_t}{\partial \kappa_z} < 0 \textit{ necessarily} \qquad \textit{and} \qquad \frac{\partial \Sigma_t}{\partial \kappa_z} > 0 \textit{ iff } \kappa_z < \hat{\kappa}$$

where $\hat{\kappa} \equiv (1-\alpha)\kappa_x - \kappa_0$.

(ii) A stronger demand externality (stronger complementarity) raises the volatility of aggregate

[9]The positive implications of these insights are explored in Angeletos and La'O (2009), while the optimality of flexible-price allocations in the presence of informational frictions is studied in Angeletos and La'O (2011).

output gaps and has a non-monotone effect on relative-price dispersion:

$$\frac{\partial \sigma_t}{\partial \alpha} < 0 \textit{ iff } \alpha > \hat{\alpha} \qquad \textit{and} \qquad \frac{\partial \Sigma_t}{\partial \alpha} > 0 \textit{ necessarily}$$

where $\hat{\alpha} \in (1/2, 1)$.

To understand part (i), note that an increase in the precision of public information induces firms and workers to reduce their reliance on their private signals, which in turn reduces the contribution of idiosyncratic noise to cross-sectional dispersion in local output choices and relative prices. At the same time, because these agents increase their reliance on noisy public news, the contribution of the noise in these news to aggregate output gaps is ambiguous. On the one hand, the increase in the precision of public information means that the level of this noise is smaller, which tends to reduce Σ_t for any given reaction to this noise. On the other hand, people are now reacting more to this information, which tends to raise Σ_t for any given level of noise. Which effect dominates depends on how large the noise is: when the precision of public information is small enough, a (marginal) increase in it ends up contributing to more volatile output gaps.

The perverse effect of public information on aggregate output gaps depends, in part, on the coordinating role that public information plays in our framework: firms and households use the available public signals, not only predict the underlying fundamentals, but also to coordinate their choices. Furthermore, as people do so, aggregate output ends up moving away from the first best.

Formalizing this last insight, part (ii) documents that a higher α raises Σ_t: the stronger the underlying aggregate demand externalities and the associated coordinating role of public information, the greater the volatility of the resulting output gaps. At the same time, one can show that the overall volatility of equilibrium output actually *falls* with α. This qualifies the precise sense in which the coordinating role of public information contributes to macroeconomic volatility: it is only the output gap, not output per se, that becomes more volatile as α increases.

This finding opens the door to the possibility that more precise public information ends up reducing welfare by raising the volatility of output gaps. Nonetheless, part (ii) of the above proposition also establishes that more precise public information helps dampen the excess dispersion in relative prices, which contribute to higher welfare. As it turns, this second, beneficial effect on relative price dispersion always dominates any potentially perverse effect on volatility. What is more, the overall positive effect on welfare is higher the higher α.

Proposition 13. *An increase in the precision of public information necessarily reduces the joint welfare losses of the volatility and dispersion in output gaps:*

$$\frac{\partial \Lambda_t}{\partial \kappa_z} < 0 \tag{2.9}$$

Furthermore, the corresponding welfare benefit of public information increases with the degree of *strategic complementarity:*

$$\frac{\partial^2 \Lambda_t}{\partial \kappa_z \partial \alpha} < 0 \tag{2.10}$$

This result contrasts sharply Morris and Shin (2002), where the social value of information turns negative when the degree of complementarity is sufficiently large. In the class of economies we are concerned with, a higher α means, not only a stronger coordinating motive and thereby more volatile output gaps, but also a lower contribution of this volatility to welfare losses relative to that of cross-sectional dispersion. As a result, the reduction in dispersion σ_t that obtains with more precise public information is more valuable when α is higher, which in turn helps this effect dominate the potentially perverse effect that public information may have on volatility Σ_t. In fact, not only is the overall effect necessarily positive, but it is also *increasing* in the degree of strategic complementarity. By contrast, the relative welfare contribution of volatility and dispersion are invariant to the degree of strategic complementarity in Morris and Shin (2002).[10] Their result therefore applies only to economies in which coordination motives are misalligned with social preferences—a scenario that does not apply to the flexible-price allocations of a canonical DSGE model.

An alternative, and perhaps more powerful, intuition to our results is also the following. Suppose there are no market distortions such as externalities, monopoly power, and labor wedges—or that the right policy instruments are in place to correct them. In this case, the equilibrium is first-best efficient when information is complete (commonly shared), but ceases to be so once information is incomplete (dispersed); the noise in the available information causes equilibrium allocations to diverge from their first-best counterparts. Nevertheless, equilibrium allocations remain *constrained efficient* in the sense that they coincide with the solution to a planning problem where the planner can dictate any allocation he wishes subject to the same resource and informational constraints as the market.[11] Note then that, by Blackwell's theorem, this planner cannot possibly be worse off with more information—if that were the case, he could simply have ignored the additional information. It is then immediate that, as long as the equilibrium attains the same allocations as this planner, equilibrium welfare *has* to increase with more precise public information.

This principle leaves outside economies where monopoly power or other market distortions create a discrepancy between equilibrium and constrained efficient allocations. Nonetheless, as long as this discrepancy takes the form of a fixed (state-invariant) wedge between the relevant marginal rates of substitution and transformation, the social value of public information is bound to remain positive. This is because a fixed wedge affects the mean level of economic activity, but does not interfere with the response of the economy to either the underlying business-cycle disturbances or the available information about them.

[10]Translating this to our context, this is the same as imposing an ad hoc welfare objective in which Σ_t and σ_t enter Λ_t symmetrically (as if α were zero), even though the equilibrium features a coordination motive (a positive α). With such an ad hoc objective, we would also have obtained that the perverse effect of public information on Σ_t dominates in some cases.

[11]See Angeletos and La'O (2009, 2011) for a definition and a proof of this kind of constrained efficiency.

Mapping this insight to our earlier welfare decomposition, this means that the aforementioned wedge implies positive first-order welfare losses ($\Delta_t < 1$), but these losses are invariant to the precision of the available public information ($\partial\Delta_t/\partial\kappa_z = 0$). The effects we documented for second-order welfare losses therefore directly translate to overall welfare: an increase in the precision of public information necessarily improves welfare, and the more so the higher the α.

Symmetric results hold if we consider the effects of private rather than public information. In particular, a higher κ_x can raise σ_t, the inefficient dispersion in relative prices, as agents pay more attention to private signals. At the same time, a higher κ_x necessarily reduces Σ_t, the volatility of aggregate output gaps, as agents shift their attention away from public news. Finally, the latter effect always dominates, guaranteeing that welfare increases with the precision of private information. Our findings can thus be summarized as follows.

Theorem 1. *Suppose the economy is hit only by shocks to technologies and preferences. Depending on whether it is public or private, more precise information can have an adverse effect on either the volatility of the output gap or the dispersion in relative prices—but not on both at the same time. All in all, welfare necessarily increases with the precision of either public or private information.*

2.6 Shocks to markups and wedges

We now shift focus from technology and preference shocks to shocks in monopoly markups and labor wedges. The distinctive characteristic of this type of shocks is that they are a source of inefficient fluctuations under complete information.

Indeed, suppose for a moment that information were complete. If a benevolent planner had access to the necessary policy instruments, he or she would completely eliminate the fluctuations generated by shocks to monopoly markups and labor wedges. With flexible prices, this could be achieved only with a state-contingent subsidy on aggregate output (or some other regulatory or tax instrument that induces the same incentives). With sticky prices, monetary policy may also play a similar role. Either way, to the extent that the available policy instruments cannot perfectly offset these shocks, the residual fluctuations generated by these shocks contribute to welfare losses relative to the first best. It is this kind of fluctuations that are the focus of this section.

To explore how incomplete information interacts with the magnitude and the welfare consequences of this kind of fluctuations, we now shut down technology and preference shocks. For expositional simplicity, we further focus on the case of markup shocks alone. The case of labor-wedge shocks is identical, modulo a change of notation/interpretation.

Let $\mu_{it} \equiv \log\left(\frac{\eta_{it}}{1-\eta_{it}}\right)$ denote the log of the local markup of island i in period t. This is given by

$$\mu_{it} = \bar{\mu}_t + \xi_{it},$$

where $\bar{\mu}_t$ captures an aggregate markup shock and ξ_{it} is a purely idiosyncratic shock. The latter is Normally distributed with mean 0 and variance σ_ξ^2, orthogonal to $\bar{\mu}_t$, and i.i.d. across islands.[12] The aggregate markup shock, on the other hand, is given by

$$\bar{\mu}_t = \chi_t + \nu_t,$$

where $\chi_t = \chi(\bar{\mu}_{t-1}, \bar{\mu}_{t-2}, ...) \equiv \mathbb{E}[\bar{\mu}_t | \bar{\mu}_{t-1}, \bar{\mu}_{t-2}, ...]$ is the component of the current markup shock that is predictable on the basis of past public information and ν_t is a Normal innovation, with mean 0 and variance $\sigma_\mu^2 \equiv 1/\kappa_\mu$, i.i.d. over time. Finally, the information structure is the same as the one we assumed for the case with productivity shocks, except that the shocks themselves have a different meaning: the private and public information are summarized in, respectively,

$$x_{it} = \bar{\mu}_t + u_{it} \qquad \text{and} \qquad z_t = \bar{\mu}_t + \varepsilon_t,$$

where u_{it} is idiosyncratic noise, with variance $\sigma_x^2 \equiv 1/\kappa_x$, while ε_t is aggregate noise, with variance $\sigma_\varepsilon^2 \equiv 1/\kappa_z$. The precision of public information is thus parameterized, once again, by κ_z.

Clearly, the first best is invariant to markup shocks (and to any information thereof). The equilibrium, however, responds to these shocks (and to any information thereof). The above log-normal structure then permits us, once again, to obtain a closed-form solution to the equilibrium allocations and the associated gaps. In particular, the complete-information equilibrium satisfies

$$\log y_{it} = \Psi' \bar{\mu}_t + \psi(\mu_{it} - \bar{\mu}_t) \qquad \text{and} \qquad \log Y_t^* = \Psi' \bar{\mu}_t$$

where $\Psi' \equiv -\frac{1}{\epsilon+\gamma} < 0$ measures the response of equilibrium output to aggregate markup shocks, while $\psi \equiv (1-\alpha)\Psi \equiv -\frac{1}{\epsilon+1/\rho} < 0$ measures the response to idiosyncratic markup shocks. The incomplete-information equilibrium, on the other hand, satisfies

$$\log y_{it} = \varphi_\mu \mu_{it} + \varphi_x x_{it} + \varphi_z z_t + \varphi_{-1} \bar{a}_{t-1},$$

where $\varphi_\mu, \varphi_x, \varphi_z$, and φ_{-1} are *negative* scalars that are determined by α, Ψ', and the information structure (see the Appendix for a detailed proof and the characterization of these coefficients). The corresponding aggregate level of output is given by

$$\log Y_t = \Phi' \bar{\mu}_t + \varphi_z \varepsilon_t + \varphi_{-1} \bar{\mu}_{t-1},$$

where $\Phi' \equiv \varphi_a + \varphi_x + \varphi_z$ is negative but higher (i.e., smaller in absolute value) than Ψ'.

Not surprisingly, local output decreases with either the local markup or any information about the aggregate markup. As a result, aggregate output decreases with either the true aggregate markup or any correlated error in people's beliefs about it. This captures a simple but important

[12]For expositional simplicity, we henceforth ignore the restriction that μ_{it} cannot be negative.

fact. Market distortions such as monopoly power and labor wedges have a powerful effect in the economy, not only due to their direct impact on individual payoffs and incentives, but also because of a powerful general-equilibrium feedback: a firm that expects the rest of the economy to experience an increase in monopoly power or other market distortions will find it optimal to reduce its own employment and production even if its own monopoly power or other local fundamentals remain unchanged. In this sense, the mere expectation of an inefficient recession may suffice for triggering an actual inefficient recession.

Turning now attention to how the available information impacts the magnitude of such inefficient fluctuations, we observe that the incompleteness of information dampens the response of aggregate output to the true aggregate markup shocks. This is similar to the property we observed earlier for the case of productivity shocks, except for one important difference. Whereas in that case this dampening effect contributed towards more volatile output gaps, now it contributes to the opposite: because the first best is now constant, dampening the response of equilibrium output to the underlying markup shocks helps stabilize the output gap.

This indicates that raising the precision of public information may now have an adverse effect on the volatility of the aggregate output gap, despite the reduction in the level of noise. At the same time, raising the precision of public information is likely to induce agents to reduce their reliance on any private information about the underlying aggregate distortions, which in turn may help reduce any inefficient dispersion in relative prices. We verify these intuitions in the next proposition, where, for comparison to the case of productivity shocks, we study the comparative statics of Σ_t and σ_t with respect to both the precision of public information and the strength of the coordinating motives.

Proposition 14. *(i) An increase in the precision of public information reduces the dispersion in relative prices and raises the volatility of the aggregate output gap:*

$$\frac{\partial \sigma_t}{\partial \kappa_z} < 0 \qquad \textit{and} \qquad \frac{\partial \Sigma_t}{\partial \kappa_z} > 0$$

(ii) A stronger aggregate demand externality (stronger complementarity) reduces both the dispersion in relative prices and the volatility in aggregate output gaps:

$$\frac{\partial \sigma_t}{\partial \alpha} < 0 \qquad \textit{and} \qquad \frac{\partial \Sigma_t}{\partial \alpha} < 0$$

The intuition behind part (i) was already discussed. To understand part (ii), note first that a stronger coordination motive (higher α) induces people to react less to private information, which dampens the impact of idiosyncratic noise on equilibrium allocations and thereby reduces the excess dispersion in relative prices. This effect is thus the same as in the case of productivity shocks. By contrast, the effect of α on the volatility of output gaps is different for essentially the same reason that the effect of public information is also different. As in the case of productivity shocks, a higher α amplifies the contribution of noise to output gaps. Yet, unlike the case of productivity shocks, a higher α helps stabilize output gaps by raising the anchoring effect of the common prior and thereby dampening the overall response of the output gap to the underlying markup shocks (formally, Ψ' falls with α). This effect is strong enough to guarantee that Σ_t falls with α. Thus, in the case of markup shocks, stronger coordination motives help stabilize output gaps.

Returning to the social value of public information, the conflicting effects on volatility and dispersion raises the possibility that the joint effect on welfare is ambiguous. The next proposition establishes that this is not the case: the adverse effect on volatility necessarily dominates.

At the same time, public information now impacts welfare, not only through volatility and dispersion, but also through the suboptimality of the mean level of output. The overall welfare effect of public information thus hinges on the comparative statics of both Λ_t and Δ_t.

Proposition 15. *An increase in the precision of public information necessarily increases the second-order welfare losses that obtain from volatility and dispersion:*

$$\frac{\partial \Lambda_t}{\partial \kappa_z} > 0$$

The joint effect of public information on volatility and dispersion is therefore the exact opposite than in the case of productivity shocks. At the same time, public information now impacts welfare, not only through volatility and dispersion, but also through the mean level of output.

Proposition 16. *An increase in the precision of public information necessarily reduces the inefficiency in the mean level of output and therefore reduces the associated first-order welfare losses:*

$$\frac{\partial (W_t^* \Delta_t)}{\partial \kappa_z} > 0$$

This finding can be explained as follows. When firms and workers face more uncertainty about the underlying aggregate shocks, the mean level of equilibrium output tends is lower. This effect is present irrespectively of whether the shocks are in preferences and technologies or in markups and labor wedges; it follows from convexity in preferences and technologies. However, the normative consequences of this effect hinge on the nature of the underlying shocks. In the case of productivity or taste shocks, the impact of uncertainty on equilibrium reflects social incentives: a benevolent planner would have reacted to the increase in uncertainty in exactly the same way as the equilibrium. This explains why δ_t is invariant to the precision of public information in the case of productivity or taste shocks. By contrast, in the case of markup or labor-wedge shocks, the impact of this uncertainty on the equilibrium is stronger than the socially optimal one. By reducing this uncertainty, more precise public information then helps reduce the associated inefficiency in the mean level of output, and thereby also to reduce first-order welfare losses.

By the envelope theorem, one may expect the gains from raising the mean level of output to be small when δ_t is close enough to 1: when the distortion is small, a marginal change in this distortion has a trivial welfare effect. This intuition suggests that the welfare losses due to volatility and dispersion are likely to dominate as long as the mean distortion is not too large. Finally, as in the case with productivity shocks, we expect the effects of private information to be symmetric to those of public. We verify these intuitions in the following theorem.

Theorem 2. *Suppose that the economy is hit only by shocks to monopoly markups and labor wedges. There exists a threshold $\hat{\delta} \in (0,1)$ such that welfare decreases with the precision of either public or private information if and only if $\delta_t > \hat{\delta}$.*

To interpret this result, recall that δ_t is the ratio of $\mathbb{E}_{t-1}[Y_t]$ to the level of output that would have maximized welfare. Hence, as long as $\delta_t < 1$, the condition $\delta_t > \hat{\delta}$ means, in effect, that the average value of the aggregate output gap is not too large. Furthermore, note that δ_t is a decreasing function of the mean value of the monopoly markup and the labor wedge. By contrast, the threshold $\hat{\delta} \in (0,1)$ is invariant to the monopoly markup and the labor wedge.[13] Thus, whether one measures the distortion by the underlying wedges or the resulting output gaps, the message remains the same: in the case of inefficient fluctuations, more precise information reduces welfare as long as the mean distortion is small enough.[14]

2.7 Macroeconomic statistics

The preceding analysis has studied the welfare effects of information under a particular specification of the information structure: we summarized the available information in exogenous signals of the shocks hitting the economy. While standard practice in the related literature, this specification is not best suited for practical questions. Consider, in particular, the following question: how does welfare depend on the quality of the macroeconomic statistics publicized by the government, or of the financial news disseminated by the public media? Unfortunately, such sources of information are *not* direct signals of the exogenous shocks hitting the economy. Rather, they are only noisy indicators of the *behavior* of other agents—they are signals of the *endogenous* state of the economy.

To capture this fact, we modify the preceding analysis as follows. In addition to (or in place of) any exogenous signals of the underlying shocks, firms and workers can now observe a noisy public indicator of the ongoing level of aggregate output. This indicator is given by

$$\omega_t = \log Y_t + \varepsilon_t \tag{2.11}$$

where $\varepsilon_t \sim N\left(0, \sigma_\omega^2\right)$ represents classical measurement error and $\sigma_\omega > 0$ parameterizes the level of this measurement error. The results are identical if we consider other indicators of aggregate economic activity, such as noisy public signals of aggregate employment and consumption, or a survey of opinions regarding any of these macroeconomic variables. The scalar $\kappa_\omega \equiv \sigma_\omega^{-2}$ can thus

[13] See the Appendix for the exact characterization of both $\hat{\delta}$ and δ_t.

[14] Clearly, this condition is automatically satisfied if the government has access to a subsidy or some other policy instrument that permits to eliminate any predictable distortion and thereby to induce $\delta_t = 1$ ($> \hat{\delta}$).

be interpreted more generally as the precision, or quality, of the available macroeconomic statistics and of any other public information regarding the endogenous state of the economy.

No matter whether the underlying shocks are in preference and technologies or in markups and labor wedges, the aforementioned macroeconomic indicator will serve, in equilibrium, as a signal of these shocks: variation in equilibrium output signals variation in the underlying fundamentals. However, the informational content of this indicator—formally, the signal to noise ratio—depends crucially on agents' behavior. It follows that, pretty much as in other rational-expectations settings (e.g., Lucas, 1972, Grossman and Stiglitz, 1980), the equilibrium must now be understood as a fixed point between the information structure and the equilibrium allocations.

In general, this fixed-point problem can be intractable. However, the combination of a power-form specification for preferences and technologies and a log-normal specification for the exogenous shocks guarantees the existence of a log-linear Rational Expectations equilibrium.

To understand this, suppose that the economy is hit only by productivity shocks. Take any allocation for which local output can be expressed as a log-linear function of the local shocks and the private information, and let φ_a and φ_x denote the corresponding sensitivities of local log output. That is, suppose

$$\log y_{it} = \varphi_a a_{it} + \varphi_x x_{it} + h_t,$$

where h_t is an arbitrary function of time, of the past aggregate productivity shocks, and of any other variable that is common knowledge as of the beginning of period t. Then, aggregate output is given by $\log Y_t = (\varphi_a + \varphi_x)\bar{a}_t + h_t$ and, since φ_a, φ_x and h_t are commonly known, observing the statistic ω_t is equivalent to observing a signal of the form

$$\omega_t' \equiv \frac{\omega_t - h_t}{\varphi_a + \varphi_x} = \bar{a}_t + \varepsilon_t', \quad \text{where} \quad \varepsilon_t' \equiv \tfrac{1}{\varphi_a + \varphi_x}\varepsilon_t.$$

This is akin to the exogenous public signal we had assumed in the preceding analysis, except for one important difference: the errors in this signal are inversely proportional to the sum $\varphi_a + \varphi_x$ and, in this sense, the precision of this signal is endogenous to the allocation under consideration. This captures a more general principle: the signal-to-noise ratio in macroeconomic statistics is pinned down by the sensitivity of equilibrium allocations to the underlying shocks to fundamentals.

Notwithstanding this important qualification, note that that this endogenous signal is Gaussian, just as the exogenous ones. It follows that the available public information can be summarized in a Gaussian sufficient statistic whose precision is given by the sum of the precisions of the exogenous and endogenous signals. In effect, this means that our preceding analysis goes through once we let

$$\kappa_z = \tilde{\kappa}_z + (\varphi_a + \varphi_x)^2 \kappa_\omega, \tag{2.12}$$

where $\tilde{\kappa}_z$ measures the precision of the exogenous public signal and κ_ω parameterizes the quality of macroeconomic statistics. We conclude that the equilibrium is now the fixed point between the allocation described in condition (2.7) and the precision of information described in (2.12).

With these observations at hand, we can now determine the social value of macroeconomic statistics simply by studying how κ_z varies with κ_ω. This is because the derivative of welfare with respect to κ_ω is equal to the derivative of welfare with respect to κ_z, which can be read off directly from Theorems 1 and 2, times the derivative of κ_z with respect to κ_ω.

Clearly, if the sum $\varphi_a + \varphi_x$ were fixed, κ_z would increase one-to-one with κ_ω. That is, if the response of equilibrium output to the underlying productivity shocks were invariant to the information structure, an increase in the quality of macroeconomic statistics would translate one-to-one to an increase in the precision of the public information. Any increase in κ_z, however, implies a reduction in the equilibrium value of φ_x: as the available public information becomes more precise, firms and households alike pay less attention to private information. By itself, this effect contributes to a lower signal-to-noise ratio in the observed macroeconomic statistic and therefore to a lower equilibrium value for κ_z: as people pay less attention to their private signals, the efficacy of social learning falls. Nonetheless, this effect is never strong enough to offset the aforementioned direct effect of κ_ω on κ_z. To see this, suppose, towards a contradiction, that the overall effect were negative. If that were the case, agents would have found it optimal to put *more* weight on their private signals, which would have implied more social learning. Both the direct and the indirect effects would then have contributed to an increase in κ_z, contradicting the original claim.

We conclude that an increase in the quality of macroeconomic statistics necessarily increases κ_z, which in turn permits us to translate all the results of the preceding analysis to the more realistic scenario in which public information regards indicators of aggregate economic activity rather than direct signals of the underlying structural shocks. The same is true for the case of markup shocks, modulo a re-interpretation of the shocks and the signals—the welfare effects of κ_z are now different, but the positive relation between κ_z and κ_ω holds true no matter whether the underlying shocks are in technologies, markups or other fundamentals.

Proposition 17. *Theorems 1 and 2 continue to hold if the precision of public information is re-interpreted as the quality of macroeconomic statistics.*

This result need not hinge on the details of the available macroeconomic statistics: we could replace the aforementioned signal of aggregate output with a signal of aggregate employment or consumption, a survey of people's forecast of these macroeconomic outcomes, or any combination of the above. This result may nevertheless depend on the absence of private forms of social learning. By this we mean the following. Suppose that, in addition to the aforementioned publicly observable macroeconomic statistics, each island also observes a noisy private signal of the output or employment level of a neighboring island, or some other private signal of the actions of other agents in the economy. This renders the *private* information that is available to each agent endogenous to the choices of other agents, pretty much as in the case of public information above. But now note that an improvement in the quality of the macroeconomic statistics may reduce the efficacy of these private forms of social learning: as each agent pays more attention to these public signals and less to his private information, the signal-to-noise ratio in the aforementioned private signals deteriorates. This deterioration, in turn, tends to have the exact opposite effect on welfare than the improvement in macroeconomic statistics: it contributes towards lower welfare in the case of productivity shocks, and towards higher welfare in the case of markup shocks.

This possibility, which is at the core of Amador and Weill (2010, 2011), qualifies the robustness of Proposition 17. Nonetheless, note that Theorems 1 and 2 characterize the welfare effects of the available information *no matter* where this information originates from. Therefore, even if one seeks to study environments that endogenize either the collection or the aggregation of information, our results remain indispensable: the welfare implications of any change in the environment that involves, directly or indirectly, a change in the available information hinges on the effects we have documented in Theorems 1 and 2.[15]

2.8 A numerical exploration

Our framework is too stylized to permit a serious quantitative analysis.[16] This qualification notwithstanding, we now consider a numerical exercise that permits us to further explore the determinants of the social value of information within the context of business cycles.

To this goal, we let the economy be hit by both productivity and markup shocks. Both shocks are log-normally distributed: $\bar{a}_t \sim \mathcal{N}(0,\sigma_a^2)$ and $\bar{\mu}_t \sim \mathcal{N}(0,\sigma_\mu^2)$, where σ_a and σ_μ parameterize the volatilities of the two shocks.[17] The local productivity and markup shocks are log-normally distributed around the corresponding aggregates and, for simplicity, comprise the entire private information of an island. Finally, the public information consists of a noisy statistic of aggregate output, defined again as in (2.11): $\omega_t = \log Y_t + \varepsilon_t$, where $\varepsilon_t \sim N\left(0,\sigma_\omega^2\right)$ is a measurement error and $\sigma_\omega > 0$ is its standard deviation.

This statistic now reveals information about *both* types of shocks: there exist scalars $\lambda_a, \lambda_\mu, \lambda_\varepsilon \in \mathbb{R}_+$ and $\lambda_0 \in \mathbb{R}$ such that, in equilibrium,

$$\omega_t = \lambda_0 + \lambda_a \bar{a}_t - \lambda_\mu \bar{\mu}_t + \lambda_\varepsilon \varepsilon_t \tag{2.13}$$

[15] For example, this observation is key to understanding how the insights of Amador and Weill (2010, 2011) may apply to the class of business-cycle economies we are interested in: those papers point out why more precise public information might slow down private learning, but it is only the analysis of our paper that informs one how this will affect output gaps, relative price dispersion, and overall welfare in a canonical business-cycle model. A similar point applies to the design of optimal policy when information is endogenous (see the discussion in the end of Section 9).

[16] Among other simplifications, we have ruled out from all forms of dynamic interdependence, such as investment and adjustment costs, thereby also abstracting from how current economic activity depends on expectations of future economic activity. Furthermore, we have assumed that the state of the economy become common knowledge by the end of each period, thereby abstracting from the rich informational dynamics that can emerge when learning takes place slowly over time. While none of these simplifications is likely to impact the essence of our theoretical insights, they are certainly relevant for quantitative questions.

[17] Adding persistence to these shocks has small effects on our results. One should only interpret σ_a and σ_μ as the volatilities of the innovations in these shocks.

where $\bar{a}_t$ and $\bar{\mu}_t$ are the aggregate productivity and markup shocks. A higher ω_t is thus interpreted partly as a signal of higher productivity and partly as a signal of lower markup. Either way, the impact on aggregate employment and output is positive: positive news about macroeconomic activity are bound to stimulate the economy irrespectively of whether these news reflect a higher aggregate productivity, a lower market distortion, or even a positive measurement error. The welfare consequences, however, crucially depend on the nature of the underlying shocks.

Parameterization. We interpret a period as a year and target a standard deviation of aggregate output growth equal to 0.02, which is consistent with US data. Together with the rest of the preference and technology parameters, this target pins down the overall volatility of the underlying shocks, but leaves undetermined the relative contribution of the two different types of shocks. We will later discuss how an estimate of the relative contribution of productivity and markup shocks can be obtained on the basis of the pertinent literature. To start with, however, we prefer to stay agnostic about this relative contribution. We thus proceed as follows. First, we measure the relative importance of the two shocks with the following statistic:

$$R \equiv \frac{(\Psi\sigma_a)^2}{(\Psi\sigma_a)^2 + (\Psi'\sigma_\mu)^2},$$

To interpret R, suppose for a moment that information were complete. Aggregate output would then be given (up to a constant) by $\log Y_t = \Psi\bar{a}_t + \Psi'\bar{\mu}_t$ and R would thus coincide with the fraction of the volatility of aggregate output that is driven by productivity shocks. We thus think of $R \in [0,1]$ as a measure of the relative importance of the two different types of shocks: a higher R means that a larger fraction of the business cycle is efficient. We then let R take any value in $[0,1]$ and, for any given value of R, we choose σ_a^2 and σ_μ^2, the volatilities of the shock innovations, so as to match both the particular value of R and the target value of the standard deviation of output.

Next, to calibrate the standard deviation of the measurement error in the signal of aggregate output, we look at the BEA releases of quarterly GDP. For a given quarter, the BEA publishes different estimates of GDP as more information becomes available. We assume that the last estimate is the true value of GDP. We set $\sigma_\omega = 0.02$, which is close to the standard deviation of the error between the first and the last release found in the literature (e.g., Fixler and Grimm, 2005).

Next, we need to parameterized the idiosyncratic noise in private information. Clearly, this is challenging, because there is no obvious way to measure private information. We partly bypass this problem by imposing that private information consists only of the local productivity and markup shocks; this restriction is motivated by the idea that, for most firms and households, private information is likely to be limited to their idiosyncratic circumstances. The precision of private information is then pinned down by the variance of the innovations in the idiosyncratic shocks.[18]

[18] Another possibility would be to calibrate the private information to match the heterogeneity observed in surveys of forecasts. This approach, however, has its own problems because it is unclear how reliable these surveys are. Given the limited scope of the numerical exercise we conduct here, we leave further exploration of this issue to future work.

Even that last option, however, is not obvious how to implement. One of the main difficulties is that different assumptions about the markup charged by a firm imply different estimates of its Total Factor Productivity (TFP). To get a rough estimate of the idiosyncratic risk in TFP, we can refer to the NBER-CES Manufacturing Database, which computes a measure of TFP for all 6-digit NAICS manufacturing industries in the US is 2005. This gives an estimate of the standard deviation of idiosyncratic TFP growth of 0.06. One may argue using a higher value to take into account the fact that the firms in the NBER-CES dataset are a more homogeneous and less volatile subset of all the firms in the US economy. Looking at sales would also suggest a bigger number for the idiosyncratic risk faced by a firm. For the markups, on the other hand, we follow the international economics literature (e.g., Tybout, 2003, Epifani and Gancia, 2010) and use the standard deviation of price-cost margins (that is, sales net of expenditures on labor and materials) as a proxy for the idiosyncratic variation in market power. Doing so yields an estimate of the standard deviation of idiosyncratic markups of about 0.1.

Based on these observations, and lacking a better alternative, we choose 0.08 as our baseline value for the standard deviation of both types of idiosyncratic shocks. Together with our restriction that private information coincides with the local shocks, this means that the noise in private information regarding the aggregate state of the economy is roughly four times as large as the noise in public information. This sounds plausible, given that the main sources of information about aggregate economic activity are likely to be public. Svensson (2005) uses a similar argument to motivate his assumption that public information is likely to be far more precise than private information. In any event, our numerical findings do not appear particularly sensitive to the value of σ_ξ.

Turning to the remaining parameters of the model, consider ϵ and γ. The former gives the inverse of the Frisch elasticity of labor supply. The latter is typically interpreted as either the inverse of the elasticity of intertemporal substitution or the coefficient of relative risk aversion. Nevertheless, as emphasized by Woodford (2003), the main role of γ in a model without capital is to control the income elasticity of labor supply.[19] Accordingly, we follow Woodford (2003) and set $\epsilon = .3$ and $\gamma = .2$. These values are such that the complete-information version of our model (which is, in effect, the basic RBC model without capital) replicates the empirical regularity that output and employment move closely together over the business cycle, while real wages are almost acyclical. We also set the average markup to 0.15, which is roughly consistent with estimates for the US economy.

The last parameter to set is the degree of strategic complementarity, α. In our model, α identifies the elasticity of individual output to aggregate output for given local fundamentals; more generally, it captures the sensitivity of actual economic activity to expectations of economic activity. Causal observation suggests that this elasticity is high. Unfortunately, however, we are not aware of any estimate of this kind of elasticity—nor is it of course clear if the particular micro-foundations we

[19]To see this, take the representative-agent version of our model. Labor supply is then given by $n_t = w_t^{1/\epsilon} C_t^{-\gamma}$, which reveals the key role played by the parameters ϵ and γ. That been said, γ also determines risk aversion, which plays evidently a role in the welfare effects of any kind of uncertainty. We return to this point below.

have considered here capture the complementarities that may be present in reality because of the far richer pattern of specialization, trade, and aggregate demand externalities.

Lacking a clear benchmark, we opt to set $\rho = 1$, which together with our choice of ϵ and γ implies a degree of strategic complementarity $\alpha = 0.6$. The motivation behind this choice is the following. Depending on how one interprets the islands of our model—as industries, classes of intermediate inputs, or even individual firms—one could argue that these goods are either complements ($\rho < 1$) or substitutes ($\rho > 1$). By setting $\rho = 1$ (Cobb-Douglas), we take the middle ground. Nonetheless, given the uncertainty we face about the appropriate calibration of the strength of aggregate demand externalities and the degree of strategic complementarity, we will later consider the sensitivity of our numerical findings to a wide range of values for α. As anticipated by our earlier theoretical results, the precise value of α is irrelevant for qualitative effects—but it is important for magnitudes.

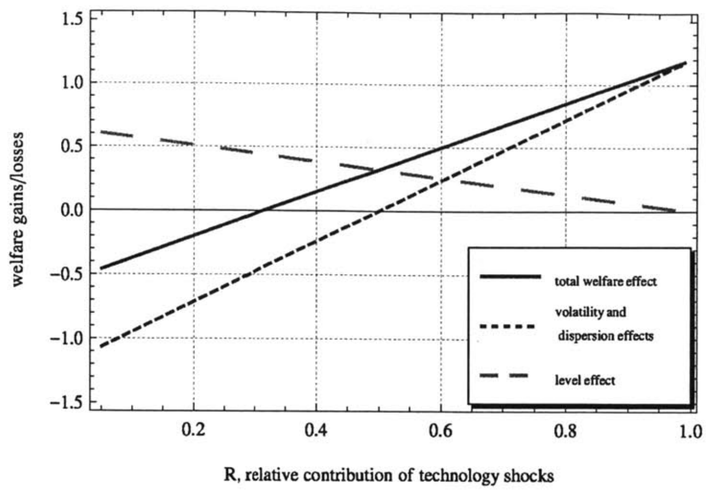
Results. With the aforementioned parameterization at hand, the exercise we conduct is to compute the welfare consequences of eliminating the measurement error in the available macroeconomic statistic. More specifically, keeping all other parameters constant, we compute the welfare gain or loss of moving the economy from $\sigma_\omega = .02$ (our baseline value for the level of noise in macroeconomic statistics) to $\sigma_\omega = 0$ (which means, in effect, perfect public information). This welfare gain or loss is computed in consumption-equivalent units (i.e., as a fraction of the mean level of consumption) and is normalized by a measure of the welfare cost of the business cycle as in Lucas (1987).[20]

Figure 1 illustrates how the welfare effects of perfecting the macroeconomic statistics vary as one varies R, the relative contribution of productivity and markup shocks. The solid line in this figure represents the total welfare effect. The other two lines decompose the total welfare effect between the effect that obtains via the impact of information volatility and dispersion (i.e., via Λ_t) and the one that obtains via the impact of information on mean level of output (i.e., via Δ_t)

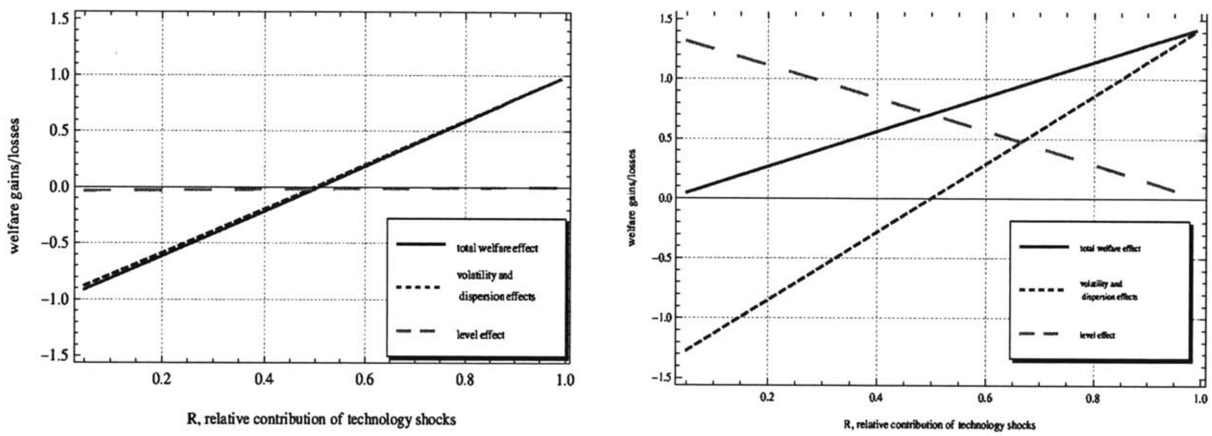
In line with Theorem 1, we see that reducing the measurement error in the macroeconomic statistics—equivalently, increasing the precision of the available public information—improves welfare when productivity shocks drive a large enough fraction of the business cycle (i.e., for high values of R). Furthermore, the welfare gains can be non trivial, at least relative to a Lucas-type measure of welfare cost of the business cycle: when productivity shocks drive the business cycle, the welfare gains of reducing the noise in macroeconomic statistics are roughly equal to the welfare gains of removing aggregate consumption risk.

As for the case where the business cycle reflects mostly variation in monopoly markups or other market distortions (i.e., for small values of R), recall from Theorem 2 that information reduces

[20] More specifically, the norm we use is the consumption-equivalent welfare gain of eliminating the risk in aggregate consumption when information is complete and fluctuations are driven by productivity shocks. We choose such a normalization for two reasons. First, by abstracting from capital and choosing a low value for γ in order to match the cyclical behavior of the economy, we have underestimated risk aversion and, in so doing, we have underestimated the welfare effects of any type of uncertainty. And second, the welfare effects of aggregate uncertainty are known to be notoriously small within the class of elementary DSGE models we are working with. Our normalization seeks to bypass these two issues by focusing on relative rather than absolute welfare effects. Enrichments of the model that increase the welfare costs of the business cycle are likely to increase as well the welfare effects we document here.



Welfare effects from eliminating the measurement error in the macroeconomic statistic as a function of R, the fraction of the business cycle that is driven by productivity shocks.



Welfare effects when the mean monopolistic distortion is reset to zero (left panel) or to a sufficiently large level (right panel).

welfare if and only if its joint detrimental effect on volatility and dispersion outweigh the beneficial one on the predictable (mean) level of economic activity. Figure 1 then reveals that this is indeed the case for the parameterization under consideration.

To clarify this issue, the dashed and dotted lines in Figure 2-1 separate the welfare effect that obtains via the impact of information on the mean level of output from the one that obtains via its impact on volatility and dispersion. The latter effect is positive if and only if $R > \frac{1}{2}$, that is, if and only if productivity shocks explains at least a half of the business-cycle volatility in output. The effect via the mean level of output, on the other hand, is positive if and only if $R < 1$, that is, as long as there are markup shocks. Furthermore, this is decreasing in R, the relative contribution of productivity shocks, and vanishes at $R = 1$. It follows that there exists a threshold $\hat{R} < \frac{1}{2}$ such that the total welfare effect is positive if and only if $R > \hat{R}$.

Clearly, the threshold $\hat{R}$ is determined by the conflict between the beneficial effect that more precise public information has on the mean level of output and the adverse effect it has on output volatility and price dispersion when markup shocks are sufficiently prevalent. As anticipated in our earlier discussion of Theorem 2, the strength of the former effect crucially depends on how big the mean distortion is: the further away output is from its optimal level, the bigger the welfare gains from a given marginal increase in output.

The role of the mean level of market distortions is illustrated Figure 2-2. The left panel in that figure repeats the exercise of Figure 1 after resetting the mean markup to zero—or, equivalently, after introducing a subsidy that undoes the mean monopolistic distortion. The impact of information on the mean level of output now has a trivial welfare effect, implying that the total welfare effect is pinned down almost entirely by the one via volatility and dispersion.

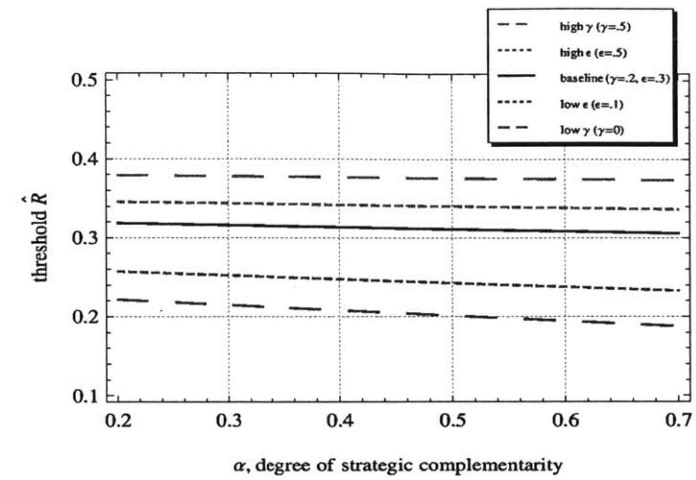
Conversely, if we let the mean distortion to be sufficient severe—which, in our numerical example, translates to a mean markup above 36%—then the impact on the mean level of output is so strong that the welfare effect becomes positive for all R. This is the case illustrated in the right panel of Figure 2. We conclude that, other things equal, a sufficiently large mean distortion suffices for information to be welfare-improving even when the business cycle is driven entirely by variation in monopoly markups and other market distortions. If one takes into account tax distortions, this scenario might actually be empirically relevant.

Turning attention to the sensitivity of our findings to other parameters, Figure 2-3 studies the comparative statics of the threshold value $\hat{R}$ at which the welfare effects of information change sign, from negative for $R < \hat{R}$ (where the markup shocks are sufficiently prevalent) to positive for $R > \hat{R}$. As evident in this figure, the threshold $\hat{R}$ tends to increase with either a higher ϵ or a higher γ. This is because an increase in either ϵ or γ tends to increases the welfare costs of volatility and dispersion relative to those of the mean distortion in output, which in turn means that the former end up playing a more important role in determining the total welfare effects of information. At the same time, we see that, for given ϵ and γ, this threshold is almost entirely invariant to α, the degree of strategic complementarity (or, equivalently, the strength of aggregate demand externalities).

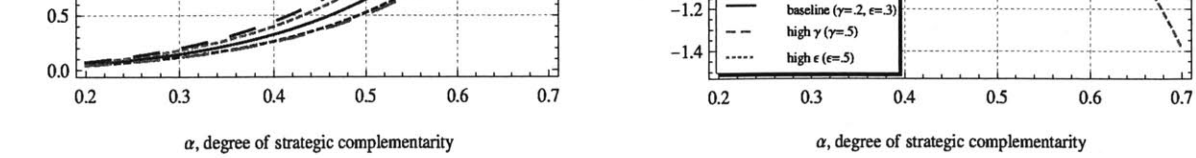
This last property underscores, once more, the contrast between our results and those of Morris and Shin (2002). In the "beauty contests" studied by them and much of the subsequent literature, the welfare effect of public information turns negative once the degree of strategic complementarity is sufficiently high and public information is sufficiently noisy. By contrast, in the class of business-cycle economies we are interested in, the degree of strategic complementarity appears to play no noticeable role in determining the *sign* of the welfare effects of information: whether information improves welfare or not depends on the relative contribution of the different shocks, not on the strength of the aggregate demand externalities and the consequent coordination motives.[21]

That been said, the degree of strategic complementarity emerges as a key determinant of the *magnitude* of the welfare effects. This is evident in Figure 2-4, which studies the sensitivity of the

[21]As it turns out, the same is true for the level of noise: the threshold $\hat{R}$ is largely invariant to σ_ω.



Sensitivity of the threshold $\hat{R}$ for the relative contribution of productivity shocks at which the welfare effects of information change sign, from negative for $R < \hat{R}$ to positive for $R > \hat{R}$.



Sensitivity of welfare effects to α, γ, and ϵ. Left panel corresponds to $R = 1$ (productivity shocks), right panel corresponds to $R = 0$ (markup shocks).

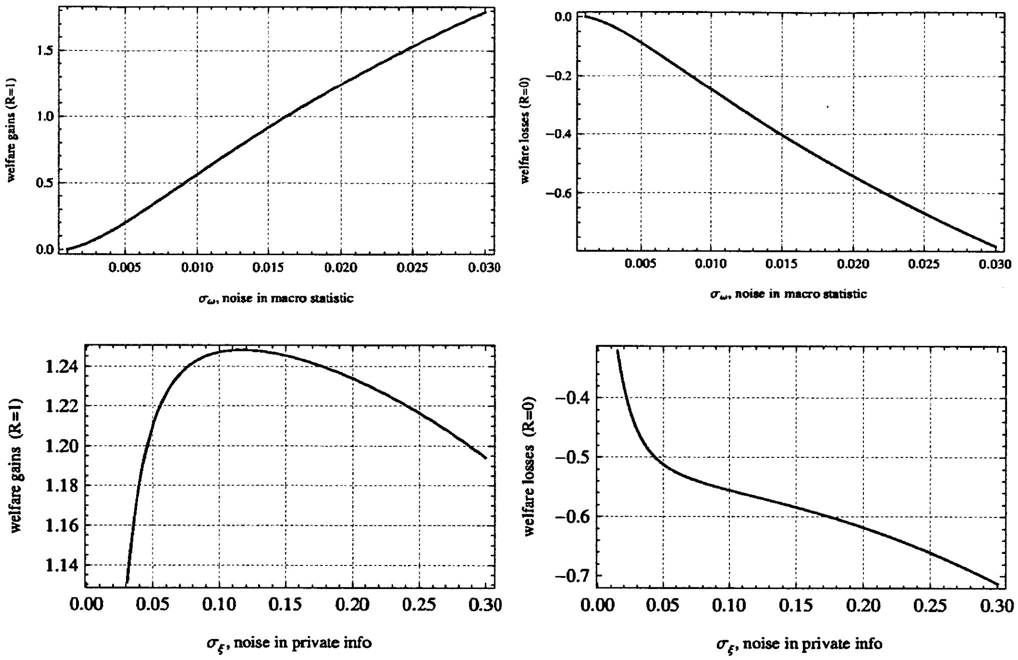
welfare effects under the two extreme scenarios where the business cycle is driven either only by productivity shocks ($R = 1$, left panel in the figure) or only by markup shocks ($R = 0$, right panel). In either case, the absolute value of the welfare effects is increasing in α: stronger coordination motives are associated with bigger welfare gains when the business cycle is driven by productivity shocks, and with bigger welfare losses when the business cycle is driven by markup shocks. Intuitively, this is because a higher α implies that firms and workers' decisions are more sensitive to the uncertainty they face about aggregate economic activity, which in turn amplifies the impact of noise on equilibrium allocations and thereby on welfare. Conversely, the welfare effects vanish as α approaches zero, for in that case expectations of aggregate economic activity—and hence any information either about the latter or about the underlying aggregate shocks—become entirely irrelevant for either equilibrium or welfare.[22]

Figure 2-4 also illustrates the sensitivity of the welfare effects to ϵ and γ. Whether the business cycle is driven by productivity shocks (left panel) or markup shocks (right panel), the welfare effects increase in absolute value when we reduce either ϵ or γ, that is, when the economy becomes more elastic to the underlying shocks. Yet, notwithstanding the uncertainty we face regarding the relevant range of values for α, the impact of ϵ and γ seems to be small relative to that of α.

Figure 2-5 concludes our numerical exploration by examining the sensitivity of the welfare effects to σ_ω and σ_ξ. Not surprisingly, raising σ_ω amplifies the welfare effect of eliminating the measurement error in macroeconomic statistics: the larger the noise in macroeconomic statistics, the larger the welfare gains of eliminating that noise in the case of productivity shocks, and the larger the welfare losses in the case of mark up shocks. The impact of σ_ξ, on the other hand, appears to be non-monotonic in the case of productivity shocks. Furthermore, the magnitude of the welfare effects is not particularly sensitive to σ_ξ as long as the latter exceeds 0.2, our baseline value for σ_ω. Since it seems unlikely that the noise in private information can be smaller than the noise in public information, this suggests that the effects we document are not particularly sensitive to this parameter for which we have very little knowledge. That been said, we would like to emphasize, once more, the limitations of our numerical exercise and the challenge of obtaining reliable estimates of either the information structure or the degree of strategic complementarity. This seems a fruitful direction for future research.

Like our theoretical exercise, our numerical exploration has so far remained agnostic about the relative contribution of the different types of shocks. In so doing, we have sought to provide a direct mapping from one's view regarding the efficiency of the business cycle to one's inference about the social value of information. Putting such priors aside, one may seek an estimate of the relative contribution of different business-cycle disturbances in the pertinent empirical literature.

[22]Note, however, that $\alpha = 0$ is a knife-edge case. As long as α is *either* positive or negative, these expectations become crucial. As a result, the welfare effects of information turn out to be qualitatively the same irrespectively of whether α is positive or negative. For instance, suppose we take the extreme case in which the good of different islands are perfect substitutes, in which case $\rho = \infty$ and $\alpha = -2/3 < 0$. The welfare effects are then qualitatively identical to those of Figure 1, although they are smaller in magnitude.



Sensitivity of welfare effects to σ_ω (top two panels) and σ_ξ (bottom two panels). In each case, the left panel corresponds to $R = 1$ (only productivity shocks), right panel corresponds to $R = 0$ (only markup shocks).

Thus consider Smets and Wouters (2007). The latter fit a DSGE model to the US economy and, among other things, estimate the contribution of different shocks to output volatility. At the one year horizon, a combination of their markup and labor-wedge shocks explains only half as much of output volatility as productivity shocks. Although their model is far richer than ours, and hence not directly comparable, their estimates suggest that we could have used $R = 2/3$ as a plausible benchmark in the numerical exercise of the preceding section.[23] Given that the threshold $\hat{R}$ that determines the sign of the welfare effects appears to be comfortably below $1/2$, we conclude that $R > \hat{R}$ seems the most likely scenario. Therefore, notwithstanding the objections one may have either about our exercise or the precise meaning of the shocks identified in estimated DSGE models, the benchmark that emerges is one where productivity shocks are sufficiently prevalent that welfare improves with more precise information.

2.9 Discussion

The key lesson of our paper is that the social value of information hinges on the nature of the underlying shocks and the efficiency of the resulting fluctuations. Withholding the release of macroeconomic statistics, practicing "constructive ambiguity", or otherwise constraining the information that

[23]Clearly, a preferred alternative would be to (i) augment their model with the information structure we have considered here, (i) estimate the augmented model on US data, and (iii) use this to quantify the welfare effects of information. Such an exercise, however, is well beyond the scope of this paper and is left for future research.

is available to the public makes sense when, and only when, a sufficiently high fraction of the business cycle is driven primarily by shocks that move the "output gap". For a plausible parameterization of our framework, this was not the case: more information was found to be welfare-improving.

Our analysis was based on the flexible-price allocations of an elementary DSGE model. As mentioned already, our focus on flexible prices was not accidental: studying the normative properties of flexible-price allocations is always the key step towards understanding the normative properties of sticky-price allocations. This principle, which is a cornerstone of the modern theory of optimal monetary policy, is also the key to the sharpness of the results we have delivered in this paper.

Needless to say, our results continue to apply if we introduce sticky prices but focus on monetary policies that replicate flexible-price allocations. Furthermore, for the case of technology or preference shocks, one can show that these policies are actually optimal (Angeletos and La'O, 2011). It follows that, for this particular case, our results extend directly to sticky prices as long as monetary policy is optimal. In the case of markup or labor-wedge shocks, on the other hand, flexible-price allocations are no more efficient and the optimal policy typically involves partial stabilization of the associated output gaps. Our results can then be interpreted as applying to the residual fluctuations that obtain once monetary policy, or other policy instruments, offset part of the underlying distortionary shocks.

These observations indicate how our results can be translated to richer settings with sticky prices. But they also underscore that this translation ought to hinge on the optimality of monetary policy—or lack thereof. More concretely, suppose that the economy is hit only by technology shocks, but, contrary to what would have been optimal, monetary policy fails to replicate flexible-price allocations. In this case, the suboptimal response of monetary policy to the underlying shocks introduces random variation in realized markups and output gaps. This opens the possibility that more precise information about the underlying productivity shocks may now be detrimental. In short, a suboptimal monetary policy might induce an otherwise innocuous productivity shock to have the same welfare implications as a distortionary markup shock.

By focusing on flexible-price allocations, we have deliberately abstracted from such confounding effects. By contrast, the suboptimality of monetary policy is playing a central role in the otherwise complementary work of Hellwig (2005) and Roca (2010). These papers assume sticky prices and study the welfare effects of information regarding exogenous shocks to the quantity of money. Clearly, monetary shocks that are unknown at the time firms set their prices cause equilibrium output to fluctuate away from the first best. To the extent that monetary policy fails to insulate the economy from such shocks, more precise public information can simply help the market do what monetary policy should have done in the first place: once these shocks become known at the time firms set their prices, prices adjust one-to-one to these shocks, guaranteeing that the shocks have no effect on real allocations and welfare. These observations help explain the results of the aforementioned two papers and the difference between their contribution and ours.

Putting aside the possibility of suboptimal monetary policy, preference and technology shocks can trigger inefficient fluctuations to the extent that market frictions impact the response of flexible-price allocations to the aforementioned shocks. For example, Blanchard and Gali (2007) show how this is a natural implication of the combination of search frictions and real-wage rigidities in the labor market. Intuitively, such rigidities make the labor wedge correlated with the technology shock. A similar argument applies to credit-market frictions: see Buera and Moll (2011) for an analysis of how different forms of credit frictions manifest in different types of cyclical wedges. In the light of our results, one would then expect the welfare effects of information to hinge on how strongly the wedges covary with the technology shocks and thereby on the severity of the market frictions. Further exploring these ideas is left for future research.

Another interesting issue emerges if the available information interferes with the ability of a policy maker to stabilize the economy. James and Lawler (2011) make a related point within the context of the Morris-Shin "beauty contest". Translating this insight in the context of business cycles hinges, once again, on the nature of the underlying shocks and the resulting "output gaps".

To recap, although our analysis abstracts from a variety of issues that may interact in intriguing ways with the question of interest, it helps resolve the apparent confusion regarding the applicability of earlier results in the literature, and lays down a clean micro-founded benchmark for understanding the welfare effects of information within the context of business cycles—a benchmark that may help guide future work on the welfare consequences of informational frictions.

Our results may indeed prove instrumental towards different exercises than the particular one we conducted in this paper. Consider, for example, how informational frictions may impact the design of optimal policy. Ongoing work in this direction includes Angeletos and La'O (2011) and Wiederholt and Paciello (2011). Although the contributions of these papers are distinct, our findings help understand the key forces operating behind some of their results. In particular, Angeletos and La'O's result that the optimal policy seeks to improve the aggregation of information through prices and macroeconomic statistics hinges on the fact that information is socially valuable when the business cycle is efficient. Similarly, Wiederholt and Paciello's result that monetary policy seeks to induce agents to pay *less* attention to markup shocks hinges on the fact that information becomes detrimental in the case of such shocks. More generally, characterizing the social value of information—which was the contribution of our paper—is a key step towards answering any normative question that involves either an exogenous or an endogenous change in the available information.

2.10 Appendix

Proof of Proposition 11. Welfare is given by

$$\mathcal{W} = \mathbb{E}\left[\sum \beta^t W_t\right]$$

where

$$W_t \equiv \mathbb{E}_{t-1}\left[\frac{Y_t^{1-\gamma}}{1-\gamma} - \tfrac{1}{1+\epsilon}\int S_{it}^n\left(\frac{y_{it}}{A_{it}}\right)^{1+\epsilon} di\right]$$

measures the period-t utility flow. Clearly, the comparative statics of welfare with respect to the information structure are pinned down by those of W_t. We henceforth focus on the latter.

To simplify the exposition, we ignore the consumption taste shocks S_{it}^c; the proof extends to this case only at the cost of more tedious derivations. Furthermore, to simplify the notation, we drop the time index t. Finally, we let $E(X)$ and $V(X)$ short-cuts for the conditional expectation and variance of a random variable X given the public information that is available at the end of period $t-1$ (which include all the period $t-1$ shocks). Similarly $Cov(X,Z)$ denotes the corresponding conditional covariance of X and Z, and $Cov(X,Z|\Theta)$ their covariance once we condition, not only the end-of-period public information of last period, but also on the current aggregate shocks Θ.

The log-normality assumption allows us to rewrite the first component of W (the one that corresponds to consumption) as follows:

$$E\left(Y^{1-\gamma}\right) = \left[E\left(Y\right)\right]^{1-\gamma}\exp\left\{-\tfrac{1}{2}\gamma\left(1-\gamma\right)V\left(\log Y\right)\right\}$$

Similarly, letting $q_i \equiv A_i^{1+\epsilon}/S_i^n$, denoting with Q the cross-sectional mean of q_i, and noting that $Y \equiv \left(\int y_i^{\frac{\rho-1}{\rho}} di\right)^{\frac{\rho}{\rho-1}}$, we can express the second component of W (the one that corresponds to leisure) as follows:

$$E\left(\int S_{it}^n\left(\frac{y_{it}}{A_{it}}\right)^{1+\epsilon} di\right) = E\left(\int \frac{y_i^{1+\epsilon}}{q_i} di\right) = \frac{\left[E\left(Y\right)\right]^{1+\epsilon}}{Q}\exp(G)$$

where

$$\begin{aligned} G \;\equiv\; & \tfrac{1}{2}\epsilon(1+\epsilon)V\left(\log Y\right) + \tfrac{1}{2}V\left(\log Q\right) - (1+\epsilon)Cov\left(\log Y, \log Q\right) \\ & + \tfrac{1}{2}\left(\epsilon + \tfrac{1}{\rho}\right)(1+\epsilon)V\left(\log y_i|\Theta\right) + \tfrac{1}{2}V\left(\log q_i|\Theta\right) - (1+\epsilon)Cov\left(\log y_i, \log q_i|\Theta\right) \end{aligned}$$

We infer that

$$W = \tfrac{1}{1-\gamma}\left[E\left(Y\right)\right]^{1-\gamma}\exp\left\{-\tfrac{1}{2}\gamma\left(1-\gamma\right)V\left(\log Y\right)\right\} - \tfrac{1}{1+\epsilon}\frac{\left[E\left(Y\right)\right]^{1+\epsilon}}{Q}\exp(G)$$

with G defined as above.

Next, let us define $\hat{Y}$ as the value of $E(Y)$ that maximizes the aforementioned expression for W, taking as given Q, G, and $V(\log Y)$. Clearly, this is given by the solution to the following condition:

$$\hat{Y}^{1-\gamma} \exp\left\{-\tfrac{1}{2}\gamma(1-\gamma)V(\log Y)\right\} = \frac{\hat{Y}^{1+\epsilon}}{Q}\exp(G) \tag{2.14}$$

We can then restate W as follows:

$$W = \left\{\tfrac{1}{1-\gamma}\left[\frac{E(Y)}{\hat{Y}}\right]^{1-\gamma} - \tfrac{1}{1+\epsilon}\left[\frac{E(Y)}{\hat{Y}}\right]^{1+\epsilon}\right\}\frac{\hat{Y}^{1+\epsilon}}{Q}\exp(G)$$

If $E(Y)$ happens to equal $\hat{Y}$, then $W = \hat{W}$, where

$$\hat{W} \equiv \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}\frac{\hat{Y}^{1+\epsilon}}{Q}\exp(G). \tag{2.15}$$

Letting

$$\delta \equiv \frac{E(Y)}{\hat{Y}} \qquad \text{and} \qquad \Delta \equiv \frac{U(\delta)-V(\delta)}{U(1)-V(1)} = \frac{\frac{1}{1-\gamma}\delta^{1-\gamma} - \frac{1}{1+\epsilon}\delta^{1+\epsilon}}{\frac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}},$$

we conclude that

$$W = \Delta\hat{W}. \tag{2.16}$$

Δ therefore identifies the wedge between actual welfare and the welfare that would have obtained if a planner had an instrument that permitted him to scale up and down the allocation under consideration by a factor δ and could choose this factor so at to maximize welfare.[24]

Next, from (2.14) we have that

$$\hat{Y} = Q^{\frac{1}{\epsilon+\gamma}}\exp\left\{-\tfrac{1}{\epsilon+\gamma}\left[G + \tfrac{1}{2}\gamma(1-\gamma)V(\log Y)\right]\right\},$$

which together with (2.15) gives

$$\hat{W} = \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}Q^{\frac{1-\gamma}{\epsilon+\gamma}}\exp\left\{G - \tfrac{1+\epsilon}{\epsilon+\gamma}\left[G + \tfrac{1}{2}\gamma(1-\gamma)V(\log Y)\right]\right\}$$

Equivalently,

$$\hat{W} = \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}Q^{\frac{1-\gamma}{\epsilon+\gamma}}\exp\left\{-\tfrac{1}{2}\tfrac{(1-\gamma)(1+\epsilon)}{\epsilon+\gamma}\hat{\Omega})\right\} \tag{2.17}$$

[24]To see this more clearly, note that Δ is strictly concave in δ_t and reaches its maximum at $\delta = 1$ when $\gamma < 1$, whereas it is strictly convex and reaches its minimum at $\delta = 1$ when $\gamma > 1$. Along with the fact that $\hat{W} > 0$ when $\gamma < 1$ but $\hat{W} < 0$ when $\gamma > 1$, this means that $\hat{W}\Delta$ is always strictly concave in δ and $\delta = 1$ is always the maximal point.

where

$$\begin{aligned}
\hat{\Omega} &\equiv \tfrac{2}{1+\epsilon}G + \gamma V(\log Y) \\
&= (\epsilon+\gamma)V(\log Y) + \tfrac{1}{1+\epsilon}V(\log Q) - 2Cov(\log Y, \log Q) \\
&\quad + \left(\epsilon + \tfrac{1}{\rho}\right) V(\log y_i|\Theta) + \tfrac{1}{1+\epsilon}V(\log q_i|\Theta) - 2Cov(\log y_i, \log q_i|\Theta)
\end{aligned}$$

Now, note that the first-best levels of output are given by the fixed-point to the following:

$$\log y_i^* = (1-\alpha)\tfrac{1}{\epsilon+\gamma}\log q_i + \alpha \log Y^*.$$

It follows that, up to some constants that we omit for notational simplicity,

$$\log Y^* = \tfrac{1}{\epsilon+\gamma}\log Q \qquad \text{and} \qquad \log y_i^* - \log Y^* = (1-\alpha)\tfrac{1}{\epsilon+\gamma}(\log q_i - \log Q)$$

Using this result towards replacing the terms in $\hat{\Omega}$ that involve q_i and Q, we get

$$\begin{aligned}
\hat{\Omega} &= (\epsilon+\gamma)V(\log Y) + \tfrac{(\epsilon+\gamma)^2}{(1+\epsilon)}V(\log Y^*) - 2(\epsilon+\gamma)Cov(\log Y, \log Y^*) \\
&\quad + \left(\epsilon + \tfrac{1}{\rho}\right) V(\log y_i|\Theta) + \tfrac{(\epsilon+\gamma)^2}{(1+\epsilon)(1-\alpha)^2}V(\log y_i^*|\Theta) - 2\tfrac{\epsilon+\gamma}{1-\alpha}Cov(\log y_i, \log y_i^*|\Theta)
\end{aligned} \tag{2.18}$$

Furthermore, the first-best level of welfare is given by

$$W^* = \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}Q^{\frac{1-\gamma}{\epsilon+\gamma}} \exp\left\{-\tfrac{1}{2}\tfrac{(1-\gamma)(1+\epsilon)}{\epsilon+\gamma}\Omega^*)\right\}$$

where Ω^* obtains from $\hat{\Omega}$ once we replace y_i and Y with, respectively, y_i^* and Y^*. We conclude that

$$\hat{W} = W^* \exp\left\{-\tfrac{1}{2}\tfrac{(1-\gamma)(1+\epsilon)}{\epsilon+\gamma}\left(\hat{\Omega} - \Omega^*\right)\right\} \tag{2.19}$$

Finally, using the definitions of $\hat{\Omega}$ and Ω^* together with the fact that $1-\alpha = \frac{\epsilon+\gamma}{\epsilon+1/\rho}$, we have

$$\begin{aligned}
\frac{\hat{\Omega} - \Omega^*}{\epsilon+\gamma} &= \{V(\log Y) + V(\log Y^*) - 2Cov(\log Y, \log Y^*)\} \\
&\quad + \frac{1}{1-\alpha}\{V(\log y_i|\Theta) + V(\log y_i^*|\Theta) - 2Cov(\log y_i, \log y_i^*|\Theta)\}
\end{aligned}$$

which together with the definitions of Σ and σ gives us

$$\frac{\hat{\Omega} - \Omega^*}{\epsilon+\gamma} = \Sigma + \frac{1}{1-\alpha}\sigma.$$

Combining this result with (2.16) and (2.19), and letting

$$\Lambda \equiv \Sigma + \frac{1}{1-\alpha}\sigma,$$

we conclude that

$$W = W^* \Delta \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\},$$

which completes the proof.

Equilibrium with productivity shocks. Suppose the equilibrium production strategy takes a log-linear form:

$$\log y_{it} = \varphi_0 + \varphi_a a_{it} + \varphi_x x_{it} + \varphi_z z_t + \varphi_{-1}\bar{a}_{t-1}, \tag{2.20}$$

for some coefficients $(\varphi_a, \varphi_x, \varphi_z, \varphi_{-1})$. Aggregate output is then given by

$$\log Y_t = \varphi_0 + X + \varphi_{-1}\bar{a}_{t-1} + (\varphi_a + \varphi_x)\,\bar{a}_t + \varphi_z z_t$$

where

$$X \equiv \frac{1}{2}\left(\frac{\rho-1}{\rho}\right) Var(\log y_{it}|\log Y_t) = \frac{1}{2}\left(\frac{\rho-1}{\rho}\right)\left[\frac{\varphi_a^2}{\kappa_\xi} + \frac{\varphi_x^2}{\kappa_x} + 2\frac{\varphi_a\varphi_x}{\kappa_x}\right]$$

adjusts for the curvature in the CES aggregator. It follows that Y_t is log-normal, with

$$\begin{aligned} \mathbb{E}_{it}\left[\log Y_t\right] &= \varphi_0 + X + \varphi_{-1}\bar{a}_{t-1} + (\varphi_a + \varphi_x)\,\mathbb{E}_{it}\left[\bar{a}_t\right] + \varphi_z z_t && (2.21)\\ Var_{it}\left[\log Y_t\right] &= (\varphi_a + \varphi_x)^2\, Var_{it}[\bar{a}_t] && (2.22) \end{aligned}$$

where, by standard Gaussian updating,

$$\begin{aligned} \mathbb{E}_{it}\left[\bar{a}_t\right] &= \frac{\kappa_a}{\kappa_a + \kappa_x + \kappa_z}\chi_t + \frac{\kappa_x}{\kappa_a + \kappa_x + \kappa_z}x_{it} + \frac{\kappa_z}{\kappa_a + \kappa_x + \kappa_z}z_t && (2.23)\\ Var_{it}[\bar{a}_t] &= \frac{1}{\kappa_a + \kappa_x + \kappa_z} && (2.24) \end{aligned}$$

Because of the log-normality of Y_t, the fixed-point condition (2.1) reduces to following:

$$\log y_{it} = (1-\alpha)(\Psi a_{it} - \Psi'\lambda) + \alpha\mathbb{E}_{it}[\log Y_t] + \Gamma \tag{2.25}$$

where $\Psi \equiv \frac{1+\epsilon}{\epsilon+\gamma} > 0$, $\Psi' \equiv \frac{1}{\epsilon+\gamma} > 0$, $\lambda \equiv -\log\left[\left(\frac{\bar{\eta}-1}{\bar{\eta}}\right)(1-\bar{\tau})\right] \approx \bar{\mu} + \bar{\tau} > 0$ is the overall distortion caused by the monopoly markup and the labor wedge (which are both constant because we are herein focusing on the case with only productivity shocks), and

$$\Gamma = \tfrac{1}{2}\alpha\left(\tfrac{1}{\rho} - \gamma\right) Var_{it}\left[\log Y_t\right] = \tfrac{1}{2}\alpha^2\left(\tfrac{1}{\rho} + \epsilon\right) Var_{it}\left[\log Y_t\right] > 0$$

Next, combining (2.25) with (2.21) and (2.23), we obtain

$$\begin{aligned}
\log y_{it} &= \Gamma - (1-\alpha)\Psi'\lambda + (1-\alpha)\,\Psi a_{it} + \alpha\left(\varphi_0 + X + \varphi_{-1}\bar{a}_{t-1} + \varphi_z z_t\right) \\
&\quad + \alpha\left(\varphi_a + \varphi_x\right)\left(\frac{\kappa_a}{\kappa_a+\kappa_x+\kappa_z}\chi_t + \frac{\kappa_x}{\kappa_0+\kappa_x+\kappa_z}x_{it} + \frac{\kappa_z}{\kappa_a+\kappa_x+\kappa_z}z_t\right)
\end{aligned}$$

For this to coincide with our initial guess in (2.20) for every event, it is necessary and sufficient that the coefficients $(\varphi_0, \varphi_a, \varphi_x, \varphi_z, \varphi_{-1})$ solve the following system:

$$\begin{aligned}
\varphi_0 &= \Gamma - (1-\alpha)\Psi'\lambda + \alpha(\varphi_0 + X) \\
\varphi_{-1} &= \alpha\varphi_{-1} + \alpha\left(\varphi_a + \varphi_x\right)\frac{\kappa_a}{\kappa_a+\kappa_x+\kappa_z} \\
\varphi_a &= (1-\alpha)\,\Psi \\
\varphi_x &= \alpha\left(\varphi_a + \varphi_x\right)\frac{\kappa_x}{\kappa_a+\kappa_x+\kappa_z} \\
\varphi_z &= \alpha\varphi_z + \alpha\left(\varphi_a + \varphi_x\right)\frac{\kappa_z}{\kappa_a+\kappa_x+\kappa_z}
\end{aligned}$$

The unique solution to this system is given by the following:

$$\varphi_a = (1-\alpha)\,\Psi > 0, \quad \varphi_x = \frac{(1-\alpha)\,\kappa_x}{\kappa_a + (1-\alpha)\,\kappa_x + \kappa_z}\alpha\Psi > 0, \quad \varphi_z = \frac{\kappa_z}{\kappa_a + (1-\alpha)\,\kappa_x + \kappa_z}\alpha\Psi > 0,$$

$$\varphi_{-1} = \frac{\kappa_a}{\kappa_a + (1-\alpha)\,\kappa_x + \kappa_z}\alpha\Psi > 0, \qquad \text{and} \qquad \varphi_0 = -\Psi'\lambda + \tfrac{1}{1-\alpha}\left(\alpha X + \Gamma\right)$$

Proof of Proposition 12. Using the characterization of the equilibrium allocation in the preceding proof along with that of the first best in (2.6), we can calculate the equilibrium values of the volatility of the aggregate output gaps and of the cross-section dispersion of local output gaps as follows:

$$\Sigma = \frac{\varphi_z^2}{\kappa_z} + \frac{(\Phi-\Psi)^2}{\kappa_a} = \frac{\alpha^2\left(\kappa_a+\kappa_z\right)}{\left((1-\alpha)\kappa_x+\kappa_z+\kappa_a\right)^2}\Psi^2$$

$$\sigma = \frac{\varphi_x^2}{\kappa_x} = \frac{\alpha^2\,(1-\alpha)^2\,\kappa_x}{\left((1-\alpha)\kappa_x+\kappa_z+\kappa_a\right)^2}\Psi^2$$

Taking the derivative of Σ with respect to the precision of public information gives

$$\frac{\partial\Sigma}{\partial\kappa_z} = \frac{(1-\alpha)\kappa_x - (\kappa_a+\kappa_z)}{\left((1-\alpha)\kappa_x+\kappa_z+\kappa_a\right)^3}\alpha^2\Psi^2,$$

which is positive if and only if $\kappa_z < (1-\alpha)\kappa_x - \kappa_0$, while taking the derivative of σ gives

$$\frac{\partial\sigma}{\partial\kappa_z} = -2\frac{\alpha^2\,(1-\alpha)^2\,\kappa_x}{\left((1-\alpha)\kappa_x+\kappa_z+\kappa_a\right)^3}\Psi^2,$$

which is necessarily negative.

Finally, taking the derivatives with respect to the degree of strategic complementarity, we obtain

$$\frac{\partial \Sigma}{\partial \alpha} = \frac{2\left(\kappa_a + \kappa_z\right)\left(\kappa_x + \kappa_z + \kappa_a\right)}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_a\right)^3} \alpha \Psi^2$$

which is necessarily positive, and

$$\frac{\partial \sigma}{\partial \alpha} = \left(\alpha^2 \kappa_x - (2\alpha - 1)\left(\kappa_x + \kappa_z + \kappa_a\right)\right) \frac{2\alpha\left(1-\alpha\right)\kappa_x \Psi^2}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_a\right)^3},$$

which is positive if and only if

$$\frac{2\alpha - 1}{\alpha^2} < \frac{\kappa_x}{\kappa_x + \kappa_z + \kappa_a}$$

Note that the RHS of the above condition is a number between 0 and 1. Next, let $g(\alpha)$ denote the LHS and note that $g:(0,1] \to \mathbb{R}$ is strictly increasing in α, with $g(1/2) = 0$ and $g(1) = 1$. It follows that there exists a unique $\hat{\alpha} \in (1/2, 1)$ such that the above condition is satisfied—and therefore σ is locally increasing in α—if and only if $\alpha < \hat{\alpha}$.

Proof of Proposition 13. Using the results from the proof of Proposition 12, we have that

$$\Lambda_t = \Sigma_t + \frac{1}{1-\alpha}\sigma_t = \frac{\alpha^2 \Psi^2}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_a\right)}$$

from which it is immediate that second-order losses are decreasing in the precision of either public or private information. (Note that this would be true even if α were negative.) Furthermore,

$$\frac{\partial^2 \Lambda_t}{\partial \kappa_z \partial \alpha} = -\frac{2\alpha\left(\kappa_x + \kappa_z + \kappa_a\right)\Psi^2}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_a\right)^3}$$

which is negative (as long as $\alpha > 0$).

Proof of Theorem 1. The distortion in the mean level of output is given by

$$\delta_t = \left[\left(\tfrac{\bar{\eta}-1}{\eta}\right)(1-\bar{\tau})\right]^{\frac{1}{\epsilon+\gamma}} < 1$$

where $\frac{\bar{\eta}-1}{\eta}$ is the monopoly wedge (the reciprocal of the markup) and $1 - \bar{\tau}$ is the labor wedge. Since δ_t, and hence also Δ_t, is invariant to the information structure, the welfare effects of public information are pinned down by comparative statics of Λ_t alone, which were established before. It follows that welfare necessarily increases with the precision of either public or private information.

Equilibrium with markup shocks. This follows very similar steps as the characterization of equilibrium in the case with productivity shocks. Suppose equilibrium output takes a log-linear form:

$$\log y_{it} = \varphi_\mu + \varphi_\mu \mu_{it} + \varphi_x x_{it} + \varphi_z z_t + \varphi_{-1}\bar{\mu}_{t-1},$$

for some coefficients $(\varphi_\mu, \varphi_x, \varphi_z, \varphi_{-1})$. This guarantees that aggregate output is log-normal, which in turn implies that the fixed-point condition (2.1) now reduces to

$$\log y_{it} = (1-\alpha)(\Psi\bar{a} - \Psi'\mu_{it}) + \alpha\mathbb{E}_{it}[\log Y_t] + \Gamma$$

where Ψ, Ψ', and Γ are defined as in the case with productivity shocks. Following similar steps as in that case, we can then show that the unique equilibrium coefficients are given by the following:

$$\varphi_\mu = -(1-\alpha)\Psi' < 0, \quad \varphi_x = -\tfrac{(1-\alpha)\kappa_x}{\kappa_\mu + (1-\alpha)\kappa_x + \kappa_z}\alpha\Psi' < 0, \quad \varphi_z = -\tfrac{\kappa_z}{\kappa_\mu + (1-\alpha)\kappa_x + \kappa_z}\alpha\Psi' < 0,$$

$$\varphi_{-1} = -\tfrac{\kappa_\mu}{\kappa_\mu + (1-\alpha)\kappa_x + \kappa_z}\alpha\Psi' > 0 \qquad \text{and} \qquad \varphi_0 = \Psi\bar{a} + \tfrac{1}{1-\alpha}(\alpha X + \Gamma)$$

Proof of Proposition 14. With only markup shocks, first-best allocations are constant. The volatility of aggregate output gaps and the dispersion of local output gaps are thus given by the following:

$$\Sigma = \frac{\varphi_z^2}{\kappa_z} + \frac{\Phi^2}{\kappa_\mu} = \frac{\alpha^2\kappa_\mu\kappa_z + ((1-\alpha)\kappa_\mu + (1-\alpha)\kappa_x + \kappa_z)^2}{\kappa_\mu(\kappa_\mu + (1-\alpha)\kappa_x + \kappa_z)^2}(\Psi')^2$$

$$\sigma = \frac{\varphi_\mu^2}{\kappa_\xi} + \frac{\varphi_x^2}{\kappa_x} + 2\frac{\varphi_\mu\varphi_x}{\kappa_x} = \frac{(1-\alpha)^2}{\kappa_\xi}(\Psi')^2 + \frac{\alpha(1-\alpha)^2(2\kappa_\mu + (2-\alpha)\kappa_x + 2\kappa_z)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2}(\Psi')^2$$

Next, taking the derivatives with respect to the precision of public information, we obtain

$$\frac{\partial\Sigma}{\partial\kappa_z} = \frac{(2+\alpha)(1-\alpha)\kappa_x + (\kappa_z + \kappa_\mu)(2-\alpha)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^3}\alpha(\Psi')^2$$

which is necessarily positive, and

$$\frac{\partial\sigma}{\partial\kappa_z} = -\frac{2(1-\alpha)^2(\kappa_\mu + \kappa_x + \kappa_z)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^3}\alpha(\Psi')^2,$$

which is necessarily negative.

Finally, taking the derivatives with respect to the degree of strategic complementarity, we obtain

$$\frac{\partial\Sigma}{\partial\alpha} = -\frac{2(1-\alpha)(\kappa_\mu + \kappa_x + \kappa_z)^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^3}(\Psi')^2$$

which is necessarily negative, and

$$\frac{\partial\sigma}{\partial\alpha} = 2(1-\alpha)(\Psi')^2 H,$$

where

$$H \equiv \frac{\alpha^2 (1-\alpha)\kappa_x^2 - \alpha(-2+3\alpha)\kappa_x((1-\alpha)\kappa_x+\kappa_z+\kappa_\mu)+(1-3\alpha)((1-\alpha)\kappa_x+\kappa_z+\kappa_\mu)^2}{((1-\alpha)\kappa_x+\kappa_z+\kappa_\mu)^3} - \frac{1}{\kappa_\xi}$$

Note that $\kappa_x \geq \kappa_\xi$, because x is the sufficient statistic of the information contained both in the own shock and in other source of private information. Letting $k \equiv \frac{\kappa_x}{\kappa_z+\kappa_\mu}$, we thus have that

$$\begin{aligned} H &\leq \frac{\alpha^2 (1-\alpha)\kappa_x^2 - \alpha(-2+3\alpha)\kappa_x((1-\alpha)\kappa_x+\kappa_z+\kappa_\mu)+(1-3\alpha)((1-\alpha)\kappa_x+\kappa_z+\kappa_\mu)^2}{((1-\alpha)\kappa_x+\kappa_z+\kappa_\mu)^3} - \frac{1}{\kappa_x} \\ &= \frac{1}{\kappa_z+\kappa_\mu}\left(\frac{-(1+k)^2}{k((1-\alpha)k+1)^3}\right) = -\frac{(1+k)^2}{\kappa_x((1-\alpha)k+1)^3} \end{aligned}$$

Since the latter is clearly negative, we conclude that $\frac{\partial\sigma}{\partial\alpha}$ is also negative.

Proof of Proposition 15. From the preceding characterization of Σ and σ, we have that

$$\Lambda = \frac{1-\alpha}{\kappa_\xi}(\Psi')^2 + \frac{(1-\alpha)\kappa_x+\kappa_z+(1-\alpha^2)\kappa_\mu}{\kappa_\mu((1-\alpha)\kappa_x+\kappa_z+\kappa_\mu)}(\Psi')^2$$

It follows that

$$\frac{\partial\Lambda}{\partial\kappa_z} = \frac{\alpha^2(\Psi')^2}{((1-\alpha)\kappa_x+\kappa_z+\kappa_\mu)^2} > 0 \tag{2.26}$$

$$\frac{\partial\Lambda}{\partial\kappa_x} = \frac{(1-\alpha)\alpha^2(\Psi')^2}{((1-\alpha)\kappa_x+\kappa_z+\kappa_\mu)^2} > 0, \tag{2.27}$$

which proves that second-order welfare losses necessarily increase with the precision of either public or private information. (Note that this would be true even if α were negative.)

Proof of Proposition 16. To study the impact of information on first-order losses Δ_t, we first need to compute δ_t. Using the equilibrium characterization and after some tedious algebra, we can show that the predictable component of aggregate output, $\mathbb{E}_{t-1}[Y_t]$, satisfies the following condition:

$$(\mathbb{E}_{t-1}[Y_t])^{1-\gamma}\exp\left(-\tfrac{1}{2}\gamma(1-\gamma)V(\log Y)\right) = \exp\left(\chi_t - \log(1-\bar{\tau}) + \tfrac{1}{2}D\right)\frac{(\mathbb{E}_{t-1}[Y_t])^{1+\epsilon}}{Q}\exp(\tilde{G})$$

where

$$\begin{aligned}
\tilde{G} &\equiv \tfrac{1}{2}\epsilon(1+\epsilon)V(\log Y) + \tfrac{1}{2}\left(\epsilon + \tfrac{1}{\rho}\right)(1+\epsilon)V(\log y_i|\Theta) \\
D &\equiv Var(\mu_{it}|\bar{\mu}_t) + 2(1+\epsilon)\,Cov(y_{it}, \mu_{it}|\bar{\mu}_t) \\
&= \frac{1}{\kappa_\xi} + \frac{1}{\kappa_\mu} + 2(1+\epsilon)\left(\frac{\varphi_\mu}{\kappa_\xi} + \frac{\varphi_x}{\kappa_x} + \frac{\varphi_\mu + \varphi_x + \varphi_z}{\kappa_\mu}\right) \\
&= \frac{1}{\kappa_\xi} + \frac{1}{\kappa_\mu} - 2(1+\epsilon)\left(\frac{1-\alpha}{\kappa_\xi} + \frac{1}{\kappa_\mu} - \frac{\alpha^2}{\kappa_\mu + (1-\alpha)\kappa_x + \kappa_z}\right)\Psi'
\end{aligned}$$

From (2.14), on the other hand, we infer that the optimal $\hat{Y}_t$ solves the following condition:

$$\hat{Y}_t^{1-\gamma}\exp\left(-\tfrac{1}{2}\gamma(1-\gamma)V(\log Y)\right) = \frac{\hat{Y}_t^{1+\epsilon}}{Q}\exp(\tilde{G})$$

Combining these results, we infer that

$$\delta_t \equiv \frac{\mathbb{E}_{t-1}[Y_t]}{\hat{Y}_t} = \exp\left[-\Psi'\left(\chi_t - \log(1-\bar{\tau}) + \tfrac{1}{2}D\right)\right],$$

and therefore

$$\frac{\partial \delta_t}{\partial \kappa_z} = -\tfrac{1}{2}\delta_t\Psi'\frac{\partial D}{\partial \kappa_z} = \frac{\alpha^2(\Psi')^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2}(1+\epsilon)\delta_t > 0 \tag{2.28}$$

$$\frac{\partial \delta_t}{\partial \kappa_x} = -\tfrac{1}{2}\delta_t\Psi'\frac{\partial D}{\partial \kappa_x} = \frac{(1-\alpha)\alpha^2(\Psi')^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2}(1+\epsilon)\delta_t > 0. \tag{2.29}$$

It follows that, as long as $\delta_t < 1$, an increase in the precision of either public or private information reduces the mean distortion—it brings δ_t closer to 1—and thereby also reduces first-order welfare losses. (Once again, this result holds true irrespectively of whether α is positive or negative.)

Proof of Theorem 2. To obtain the overall welfare effect, recall that welfare is given by

$$W_t = W_t^*\Delta_t\exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda_t\right\}$$

Consider first the case of public information. From the above, we have that

$$\frac{\partial W_t}{\partial \kappa_z} = W_t^*\exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda_t\right\}\left(\frac{\partial \Delta_t}{\partial \delta_t}\frac{\partial \delta_t}{\partial \kappa_z} - \tfrac{1}{2}\Delta_t(1+\epsilon)(1-\gamma)\frac{\partial \Lambda_t}{\partial \kappa_z}\right)$$

From (2.26) and (2.28), we have that

$$\frac{\partial \delta_t}{\partial \kappa_z} = \frac{\partial \Lambda_t}{\partial \kappa_z}(1+\epsilon)\delta_t$$

It follows that

$$\frac{\partial W_t}{\partial \kappa_z} = W_t^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda_t\right\} \frac{\partial \Lambda_t}{\partial \kappa_z} H_t$$

where

$$\begin{aligned} H_t &\equiv \frac{\partial \Delta_t}{\partial \delta_t}(1+\epsilon)\delta_t - \tfrac{1}{2}(1-\gamma)(1+\epsilon)\Delta_t \\ &= \tfrac{(1-\gamma)(1+\epsilon)}{2(\epsilon+\gamma)}\left[2(1+\epsilon)\left(\delta_t^{1-\gamma} - \delta_t^{1+\epsilon}\right) - \left((1+\epsilon)\delta_t^{1-\gamma} - (1-\gamma)\delta_t^{1+\epsilon}\right)\right] \\ &= \tfrac{(1-\gamma)(1+\epsilon)}{2(\epsilon+\gamma)}\left[(1+\epsilon)\delta_t^{1-\gamma} - (1+2\epsilon+\gamma)\delta_t^{1+\epsilon}\right], \end{aligned}$$

and therefore

$$\frac{\partial W_t}{\partial \kappa_z} = \tfrac{(1-\gamma)(1+\epsilon)}{2(\epsilon+\gamma)} W_t^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda_t\right\} \frac{\partial \Lambda_t}{\partial \kappa_z}\delta_t \left[(1+\epsilon) - (1+2\epsilon+\gamma)\delta_t^{\epsilon+\gamma}\right]$$

Note then that the sign of W_t^* is the same as that of $(1-\gamma)$, which together with the facts that $\frac{\partial \Lambda_t}{\partial \kappa_z} > 0$ and $\delta_t > 0$ implies that the sign of $\frac{\partial W_t}{\partial \kappa_z}$ is the same as the sign of $(1+\epsilon) - (1+2\epsilon+\gamma)\delta_t^{\epsilon+\gamma}$. We conclude that

$$\frac{\partial W_t}{\partial \kappa_z} < 0 \quad \text{iff} \quad \delta_t > \hat{\delta},$$

where

$$\hat{\delta} \equiv \left(\frac{1+\epsilon}{1+2\epsilon+\gamma}\right)^{\frac{1}{\epsilon+\gamma}} \in (0,1).$$

Consider next the case of private information. From (2.27) and (2.29), we have that

$$\frac{\partial \delta_t}{\partial \kappa_x} = \frac{\partial \Lambda_t}{\partial \kappa_x}(1+\epsilon)\delta_t,$$

pretty much as in the case of public information. It follows that

$$\begin{aligned} \frac{\partial W_t}{\partial \kappa_x} &= W_t^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda_t\right\}\left(\frac{\partial \Delta_t}{\partial \delta_t}\frac{\partial \delta_t}{\partial \kappa_x} - \tfrac{1}{2}\Delta_t(1+\epsilon)(1-\gamma)\frac{\partial \Lambda_t}{\partial \kappa_x}\right) \\ &= W_t^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda_t\right\}\frac{\partial \Lambda_t}{\partial \kappa_x} H_t \end{aligned}$$

where H_t is defined as before. By direct implication,

$$\frac{\partial W_t}{\partial \kappa_x} < 0 \quad \text{iff} \quad \delta_t > \hat{\delta},$$

where $\hat{\delta}$ is the same threshold as the one in the case of public information.

Proof of Proposition 17. Consider the case where the economy is hit only by productivity shocks. From the analysis in the main text, we have that the equilibrium must satisfy

$$\kappa_z = \tilde{\kappa}_z + (\varphi_a + \varphi_x)^2 \kappa_\omega,$$

From the analysis of the equilibrium with productivity shocks in the beginning of this appendix, we have that

$$\varphi_a + \varphi_x = \frac{\kappa_a + \kappa_x + \kappa_z}{\kappa_a + (1-\alpha)\,\kappa_x + \kappa_z}(1-\alpha)\Psi,$$

Combining, we conclude that the equilibrium value of κ_z is pinned down by the following fixed point:

$$\kappa_z = F(\kappa_z)$$

where

$$F(\kappa_z) \equiv \tilde{\kappa}_z + \left(\frac{\kappa_a + \kappa_x + \kappa_z}{\kappa_a + (1-\alpha)\,\kappa_x + \kappa_z}\right)^2 (1-\alpha)^2 \Psi^2 \kappa_\omega$$

Note that F is continuous and decreasing in κ_z, with $F(0) > 0$. It follows that there exists a unique solution to $\kappa_z = F(\kappa_z)$, which means that the equilibrium is unique. Furthermore, since F is increasing in κ_ω, the equilibrium value of κ_z is also increasing in κ_ω. Along with the facts that κ_ω impacts welfare only through κ_z and that welfare is increasing in κ_z, this proves that welfare is increasing in κ_ω.

The same arguments apply to the case of markup shocks, modulo a change in notation/interpretation.

Chapter 3

Liquidity Insurance with Market Information

3.1 Introduction

Asymmetric information is a common problem in credit markets where firms borrow capital to finance their investment opportunities and to face unexpected events. Should a particular investment be undertaken? Should credit be extended to a firm that might go bankrupt? If lenders and borrowers do not share the same information, the informed agents are in a position to exploit their advantage and gain at the expense of the other agents. As a result, many profitable investment opportunities are not undertaken and many viable firms cannot find enough capital to continue their business.

Banks and other lenders deal with the issue that borrowers may be better informed by devoting considerable resources to analyze all sorts of information to screen potential borrowers. This task often continues during the whole life of the loan when banks monitor borrowers to make sure loans are repaid. This is a costly process and it is often impossible or not optimal for the lender to learn the quality of the borrower perfectly. When the problem of asymmetric information is not fully resolved, financial marktes are negatively affected and firms cannot raise the necessary capital.

Financial markets are themselves a potential source of information. Many firms are traded daily on the stock market and derivatives – such as credit default swaps – provide timely information on the health of a firm. If asset prices offer valuable information, then intuitively they should be taken into account in lending decisions. For example, a bank may consider to lend to a firm only if its stock price is above a certain threshold or if the price of a CDS signals that the probability of bankruptcy is low.

There are, however, two potential problems with this. First, if we consider all the information that a bank acquires before lending to a firm, it seems that the usefulness of asset prices as a source of information is quite limited. In other words, if a bank can screen potential borrowers, then why would it use the information contained in asset prices? This is even more important if we think of asset prices as being subject also to fluctuations that are not related to the value of the firm. Can it be optimal for a bank or a firm to let the success of an investment depend on non-fundamental fluctuations of asset prices?

Secondly, asset prices respond endogenously to the agents' decisions. Stock prices, for example, depend on expectations about the firm's future dividends. Thus, whether a firm finds capital to pursue a new investment opportunity or to cover an unexpected cost will certainly affect its stock price. How can then a bank use the information contained in asset prices when these prices are themselves affected by the decision of the bank?

In this paper, I study how asymmetric information between lenders and borrowers – such as small entrepreneurs, managers, or controlling shareholders of big corporations – affects liquidity management and investment choices. I then investigate how market signals – such as stock prices – can mitigate the distortions arising from the asymmetry of information. Finally, I show that the endogeneity of asset prices may limit the ability of investors to provide liquidity in an optimal way.

The building block is the Holmstrom and Tirole (1998) model of liquidity insurance. Demand for liquidity arises from the fact that entrepreneurs face unexpected costs, but lack the commitment to pledge the total NPV of their firm to investors. One partial solution is to contract with investors in advance and secure liquidity before the realization of the unexpected costs. However, more insurance against liquidity shocks makes the business less profitable for investors who invest less capital in the firm. One advantage of this model is that, while it departs from the Arrow-Debreu paradigm assumed in Modigliani and Miller (1958) in a very simple way, it is enough to generate a demand for liquidity and a trade-off between the capital invested in a firm and the amount of insurance against liquidity shocks. Also, the simple structure of this model delivers tractability when I allow for endogenous asset prices in section 3.5.

I show that investors discriminate between the different types of borrowers by offering them menus of contracts that differ for the amount of capital invested in the firm and the amount of liquidity insurance. In particular, relative to the symmetric information benchmark, good borrowers receive a distorted contract with less insurance. Since good borrowers are more productive, they prefer the risk of running out of cash (that is, they prefer less insurance) for a higher investment in the firm.

Asset prices can mitigate the asymmetric information problem and reduce the distortions of the contracts. These prices, however, are observed after the investment is undertaken and, thus, after lenders and borrowers sign the contract. On the equilibrium path, investors offer different contracts to the the different types of borrowers and, therefore, the asymmetric information is resolved *before* investors observe asset prices. Thus, on the equilibrium path investors do not use financial markets to get information about the borrowers. However, even if prices are observed ex-post, investors still find it optimal to condition liquidity insurance on asset prices. By doing so, they can provide borrowers with better incentives for truth-telling and offer less distorted contracts. Intuitively, if borrowers expect lenders to learn from financial markets at some point in the future, they will have a greater incentive to disclose information at the contracting stage. Conditioning a contract on asset prices, however, comes with the cost of exposing the firm to the non-fundamental fluctuations in financial markets. Thus, a firm may run out of liquidity only because its price is too low for some non-fundamental reason.

Related Literature. Firms in this paper will be subject to unexpected costs and will want to insure against them. This model builds mainly on Holmstrom and Tirole (1998) where the lack of commitment on the entrepreneurs' side prevents them to borrow as much as they would like. Several other theories have been proposed in corporate finance to explain why firms may be willing to buy insurance against adverse scenarios. In von Thadden (2004) the presence of adverse selection may limit the borrowing capacity of a firm that hasn't planned liquidity in advance. "Signal-jamming," whereby competitors prey on each other to make rivals look financially weak, is another reason for planning funding in advance (Bolton and Scharfstein (1990) and Fudenberg and Tirole (1985)).

This paper is also related to the literature of insurance under asymmetric information. In their seminal paper, Rothschild and Stiglitz (1976) study a model of insurance markets where insurers do not know the type of their customers. Similarly, in this paper lenders are uncertain about the profitability of the borrorwers' investment opportunities and they offer a menu of contracts to screen the different types of borrowers. Apart from the role of asset prices, this paper differs from Rothschild and Stiglitz (1976) for another reason. For most of the paper, I assume that lenders and borrowers meet without any of them knowing the quality of the firm. Lenders then offer a menu of contracts to the borrowers. The latter then learn the quality of their firms and select a contract in the menu. This assumption on how agents acquire information over time greatly simplifies the analysis without affecting the main message of the paper, as I show in appendix 3.8.

There is a vast literature that studies how investors use market signals to acquire information about the quality of the borrowers. Some papers also study how the endogeneity of asset prices affects the amount of information they provide. Angeletos et al. (2010) consider entrepreneurs who make an investment whose profitability depends on the future price of capital. In turn, the price of capital depends on the investment decisions of all the entrepreneurs in the economy. In this paper, the mechanism behind the feedback is different: investors use asset prices only insofar as they want to discriminate between different types of borrowers. Bond and Goldstein (2011) study how government intervention can affect the informativeness of asset prices by affecting the trading incentives of speculators. In Bond et al. (2010) some agents learn the value of a firm from asset prices and then take actions that, in turn, affect the value of the firm. One important difference with these models is that that I assume asymmetric information between investors and entrepreneurs. Also, on equilibrium investors screen the different types of borrrowers (and, therefore, learn their types), but observe asset prices only *after* the contract is signed. Therefore, they do not acquire any information from asset prices, but use them only to relax the ex-ante screening problem.

For the particular way in which market signals affect the equilibrium allocations, this paper is also related to some papers in the mechanism design literature that study how a principal can use ex-post signals to provide better incentives to the agent (Riordan and Sappington (1988), Bose and Zhao (2007)). They show that, even if in equilibrium the signal does not reveal extra information to the principal, the latter still finds it optimal to condition the contract on the signal. The reason is that, by conditioning the terms of the contract on a signal that is informative about the agent's true type, the principal alleviates the distortions coming from the asymmetry of information. Similarly, in my model investors will offer contracts that are conditional on market signals to provide entrepreneurs with better incentives to reveal their type. Finally, this paper is also related to the large literature in macroeconomics that studies the implications of the feedback between investment decisions and asset prices arising from financial frictions (Bernanke and Gertler (1989) and Kiyotaki and Moore (1997))[1].

The paper is organized as follows. In section 3.2 I present the model and define the equilibrium. In section 3.3 I solve the symmetric information benchmark. Section 3.4 introduces asymmetric information and derives the implications for the investment and liquidity insurance decisions. In section 3.5 I solve the full model with asymmetric information and endogenous asset prices. In this section, I also perform some comparative static exercises to show how a decrease in the informativeness of asset prices affect the equilibrium. Section 3.6 contains the concluding remarks. Finally, appendix 3.8 shows that the main conclusions of the model do not change when I make different assumptions about the asymmetry of information between borrowers and lenders.

3.2 The Model

The economy lasts for three periods $t = 0, 1, 2$ and is populated by three types of agents.

First, there are entrepreneurs who are endowed with a project and with A units of cash. Entrepreneurs are risk neutral and consume only at time 2. A project is a constant return technology that generates a random return in period 2. The project can be of two possible types, θ^H with probability α and θ^L with probability $1-\alpha$, with $\theta^H > \theta^L$. Let $\tilde{y}(\theta)$ be the return per unit of investment generated at time 2 by a project of type θ. For simplicity, I will assume that $\tilde{y}(\theta)$ can have only two possible realizations

$$\tilde{y}(\theta) = \begin{cases} R & \text{w.p. } \theta \\ 0 & \text{w.p. } 1-\theta \end{cases}$$

Let Θ denote the space of possible types, that is, $\Theta = \{\theta^L, \theta^H\}$. This payoff captures the idea that

[1]See also Acemoglu and Zilibotti (1997), Lamont (1995), Carlstrom and Fuerst (1997), Aghion et al. (1999), Rampini (2004), Cooley et al. (2004), Guerrieri and Lorenzoni (2009), and references in Bernanke et al. (1999).

in period 2 a high type project generates a higher return than a low type project.

Entrepreneurs are assumed to be risk neutral, hence without other assumptions there would be no scope for insurance in this model. The assumption of risk neutrality in period 2 is appealing since I will think of "entrepreneurs" mostly as managers or control shareholders of big firms or banks. Arguably, these agents can diversify away their exposure to the idiosyncratic risk of the firm they control, and are effectively risk neutral.

In order to produce the final return, the project requires an initial investment I at time 0, and a random reinvestment $\rho \sim f_\theta(\cdot)$ per unit of initial investment at time 1. The initial investment I determines the scale at which the project is operated. Thus, if carried on to period 2, a project of type θ and scale I will generate a random return of $\tilde{y}(\theta)I$. On the contrary, if the liquidity shock is not covered, the project is liquidated and generates a return $l(\theta)I$, with $l(\theta) < R\theta$.

I will often refer to the random reinvestment need at time 1 as a "liquidity shock". I assume that the distribution $f_\theta(\cdot)$ satisfies the monotone likelihood property (MLP) with respect to θ. Formally, I assume that $f_L(\rho)/f_H(\rho)$ is decreasing in ρ. The MLP assumption captures the idea that a high type project is more productive also at time 1. In other words, on average, a high type project generates a higher final payoff and is cheaper to complete (expected liquidity shock in period 1 is lower).

The random reinvestment need can have different interpretations. One way to think about it is that the firm at time 1 experiences a cost overrun which has to be covered in order to continue the project. Cost overruns are not so uncommon in reality. Altough hard to estimate, some studies show that big corporations, in particular in the construction and technology business, experience significant overrun costs during the life of the project.

A more general interpretation for ρ is that the project generates random income at time 1, but requires an extra, random investment to be made at time 1. Under this interpretation, ρ represents the difference between the random income and the random cost in period 1. In this case ρ can either be positive, the project generates positive net income and can be continued, or negative, the project requires an extra injection of cash to continue. For example, banks may invest in assets that turn out to be worth less than expected and need to raise funds to continue their normal activities.

In what follows, and in the numerical simulations, I will think of ρ as an overrun cost, but both interpretations are possible.

Finally, I assume that all the projects are independent and identically distributed. Independence implies that in the economy there will be exactly α entrepreneurs of type θ^H and $1-\alpha$ entrepreneurs of type θ^L. The i.i.d. assumption also implies that $f_\theta(\cdot)$ will be the cross section of liquidity shocks per unit of investment for projects of type θ.

The following assumption guarantees that it is optimal to undertake both types of projects.

Assumption 3. *Both projects have positive NPV:*

$$\max_{\tilde{\rho}} \left\{ R\theta F_{\theta}(\tilde{\rho}) - 1 - \int_{0}^{\tilde{\rho}} \rho f_{\theta}(\rho)\, d\rho \right\} > 0, \quad \theta \in \Theta$$

The maximand is the return from investing 1 unit of wealth in a project of type θ. Investing 1 unit of wealth doesn't have to yield a positve NPV. In fact, if the project is continued for all values of ρ, then the investment is likely to deliver a negative NPV. However, the assumption requires that, if the threshold above which the project is terminated is chosen optimally (as it will be the case in the optimal contract), the project yields a positive NPV.

Finally, I assume that an entrepreneur can run away with a fraction $R - \rho_0$ of the final outcome. This assumption implies that only a fraction ρ_0 of the final return can be pledged to investors. On the other hand, the liquidation value of the firm $l(\theta) I$ is fully pledgeable to investors. Hence, the moral hazard arises only if the project is continued. Different assumptions about the pledgeability of the liquidation value would not change the main message of the model. The form of limited commitment assumed here is closely related to Holmstrom and Tirole (1998) and will imply that entrepreneurs will find it optimal to hedge against liquidity shocks.

The second type of agents in the economy are investors. Investors are also risk-neutral and care only about time 2 consumption. They do not have a project, but they have a large endowment. A single investor can finance one or more different projects. Because projects are i.i.d. investors can pool different projects together and generate deterministic cash flows (this is, however, not important given the assumption of risk-neutrality). Free-entry in the investors' business will drive their profits to zero. Finally, I assume that investors can transfer consumption across different periods at the fixed risk-free rate r.[2]

The third group of agents are the traders. I assume that in the economy there is a betting market where, for each firm, a security in zero-net supply is traded. Traders do not have a project, but can trade in the betting market. Each trader is endowed with wealth W and has access to the same storage technology as investors. Traders are risk neutral and maximize final wealth. For simplicity, I assume that the payoff of the security traded in the market is $\tilde{y}(\theta)$ if the project is continued and $l(\theta)$ if the project is liquidated. The specific payoff of the security is not important for the conclusions of the model. Of course, it is key that the payoff depends on the type θ.

Traders receive a signal $\tilde{s}_{\theta}$ at time 1 about each firm in the economy. The signal contains information about the type θ of the firm. Traders use this information to decide how much money to invest in the security. It is irrelevant whether different groups of traders specialize in different firms and receive information only about those firms, or whether every trader observes the signal

[2]The assumption that investors have a large endowment, together with the availability of a storage technology, simplifies the analysis without affecting the main results of the paper. Introducing consumers who elastically supply funds at time 0 and allowing the risk-free rate to adjust to equate demand and supply is relatevely straightforward.

about every firm in the economy.

Also, since it simplifies things while leaving the main message of the model unchanged, I am going to assume that the signal $\tilde{s}_\theta$ is perfectly revealing of the type of the firm. Formally, I will assume that $\tilde{s}_\theta = \theta$.

Finally, before making his portfolio decision, every trader observes the liquidity shock ρ that has hit the firm.

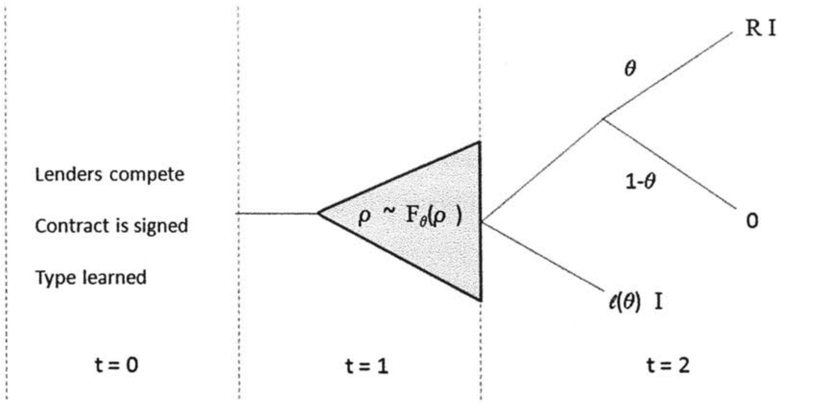
Traders submit demand schedule and the betting market is modelled as a Walrasian market. Let $D_i(\theta, p, \rho)$ be the quantity of the security demanded by trader i, when he knows that the type of the firm is θ, he observes a liquidity shock ρ, and the price of the security is p. In order to buy a quantity $D_i(\theta, p, \rho)$ of the security, trader i has to pay a transaction cost κ proportional to the square of the quantity demanded. Since all the traders have access to the same information, in equilibrium they will all demand the same quantity and I can drop the subscript i from $D_i(\theta, p, \rho)$.

The supply of this security is random and equal to $u \sim N\left(0, \sigma_u^2\right)$.

The combination of the random supply u and the transaction cost κ implies that the price will not be fully revealing in equilibrium. The distribution of the asset price will be determined in equilibrium given the choices of investors and entrepreneurs. I denote with $\phi_P(\cdot|\theta, \rho)$ the probability density function of the equilibrium distribution of the asset price if the entrepreneur is of type θ and a liquidity shock ρ is observed. Of course, the price is informative because the equilibrium price distribution is a function of the true type of the entrepreneur. For the rest of the paper, let me set $\kappa = 1$ to simplify the notation.

Timing and Information. The timing of the model is as follows. Each entrepreneur meets an investor and signs a contract specifying, among other things, the initial investment I and, for each liquidity shock ρ, whether the firm will be continued or liquidated. For most of the paper, except for section 3.8, I will assume that entrepreneurs learn their types only *after* they have signed the contract with the investor. This assumption greatly simplifies the analysis without altering the main conclusion. When entrepreneurs learn their types only after the contracting stage, the notion and the properties of the equilibrium are standard. Investors offer contracts that maximize entrepreneurs' ex-ante welfare subject to a zero-profit condition and the incentive compatibility constraints for the two types of entrepreneurs. On the other hand, if entrepreneurs knew their types before signing the contract, then the combination of adverse selection and competition among investors would make it much harder to define an equilibrium and prove its existence. The following graph summarizes the timing of the model.

I will come back to these issues in section 3.8 where I let entrepreneurs learn their types before contracting takes place. The reason for doing this is twofold. First, I can show that the analysis here is robust to this assumption. But, more importantly, this alternative assumption allows me to make an important point on how investors use the ex-post information contained in asset prices.



3.2.1 Contracts

For a project to be completed, an investor and an entrepreneur have to meet at time 0 and agree on an initial investment I and on a contingent continuation rule that specifies, for each realization of ρ, whether the project is continued or liquidated. Finally, either if the output is realized or the project is liquidated, the investor and the entrepreneur have to agree on how to share the surplus. All these elements are part of the definition of a contract.

Definition. *A contract C is a tuple $\left[I, \chi, \bar{c}^e, \underline{c}^e, c^e_L, \bar{c}^i, \underline{c}^i, c^i_L\right]$, which specifies:*

1. *payments $\bar{c}^e : \mathbb{R} \times \varrho \times \Omega \to \mathbb{R}_+$, $\underline{c}^e : \mathbb{R} \times \varrho \times \Omega \to \mathbb{R}_+$, and $c^e_L : \mathbb{R} \times \varrho \to \mathbb{R}_+$ to the entrepreneur after observing a price p and a liquidity shock ρ, in the case the project is succesful, completed but unsucessful, and liquidated, respectively;*
2. *payments to the investor $\bar{c}^i : \mathbb{R} \times \varrho \times \Omega \to \mathbb{R}$, $\underline{c}^i : \mathbb{R} \times \varrho \times \Omega \to \mathbb{R}$, and $c^i_L : \mathbb{R} \times \varrho \to \mathbb{R}$;*
3. *an initial investment $I \in \mathbb{R}_+$ in the project;*
4. *a continuation rule $\chi : \mathbb{R} \times \varrho \to \{0, 1\}$ which, for each value of p and ρ, is equal to 1 whenever the project is continued.*

Remember that at the time when an investor and an entrepreneur meet, none of them knows the type of the project. However, after the contract is signed the entrepreneur learns his type. I then allow investors to offer menus of contracts to the entrepreneur. When the entrepreneur learns his type, he will select the contract in the menu that maximizes his return.

Let $\mathbb{C}$ be the space of all possible contracts. A *menu of contracts* is a subset of $\mathbb{C}$.

By assumption, an entrepreneur can always guarantee for himself a fraction $R - \rho_0$ of the final outcome. If an investor were to offer a menu of contracts $\mathcal{C}$ with a payment to the entrepreneur less than $(R - \rho_0) I$ in the case of success, the entrepreneur would make up for the difference by stealing the output. Therefore, every menu of contracts $\mathcal{C}$ offered in equilibrium has to be such that $\forall C \in \mathcal{C}$

$$\bar{c}^e (p, \rho, R) \geq (R - \rho_0) I \tag{3.1}$$

To simplify notation, let $\mathbb{C}^{MH}$ be the subset of $\mathbb{C}$ containing only the contracts that satisfy (3.1).

3.2.2 Payoffs

Given a menu of contracts $\mathcal{C} \subseteq \mathbb{C}$, suppose that the entrepreneur selects the contract $C \in \mathcal{C}$ when he learns that his type is θ. We can compute the entrepreneur's expected utility *conditional* on being of type θ as follows:

$$U(C;\theta) = \mathbb{E}\left[\theta\chi(p,\rho)\,\bar{c}^e(p,\rho) + (1-\theta)\,\chi(p,\rho)\,\underline{c}^e(p,\rho) + (1-\chi(p,\rho))\,c^e_L(p,\rho)\,|\theta\right]$$

where the expectation is taken over the realizations of p, ρ and the final return $y(\theta)$. From now on I will denote by C_θ the contract in the menu $\mathcal{C}$ selected by an entrepreneur of type θ and I will add a substript θ to all the objects in C_θ.

With this notation I can define the expected utility of an entrepreneur who chooses a menu of contract $\mathcal{C}$ *before* learning his type:

$$U(\mathcal{C}) = \alpha U(C_L;\theta_L) + (1-\alpha)\,U(C_H;\theta_H),$$

where

$$C_\theta \in \arg\max_{C\in\mathcal{C}} U(C;\theta),$$

is the contract in the menu that is chosen by type θ.

Similarly, denote by $\pi(C;\theta)$ the expected profits of an investor who has offered a menu of contracts $\mathcal{C} \subseteq \mathbb{C}$ to an entrepreneur who turns out to be of type θ and chooses contract $C \in \mathcal{C}$:

$$\pi(C;\theta) = \mathbb{E}\left[\theta\chi(p,\rho)\,\bar{c}^i(p,\rho) + (1-\theta)\,\chi(p,\rho)\,\underline{c}^i(p,\rho) + (1-\chi(p,\rho))\,c^i_L(p,\rho)\right] - \mathbb{E}\left[1+\rho\chi(p,\rho)\right]I$$

As before, the expectation is over the realizations of p, ρ and the final return $y(\theta)$. The last term in $\pi(C;\theta)$ represents the expected costs for the investor when contract C is chosen. This is the sum of two components: the initial investment outlay I, and the expected cost of reinvestment multiplied by the indicator function of whether reivestment occurs, $\chi(p,\rho)$.

Note that, except for the symmetric information benchmark in section 3.3, the investor can learn the type of the entrepreneur only by looking at the choice that the entrepreneur makes. Thus, the investor doesn't know $\pi(C;\theta)$, even though he may be able to learn it in equilibrium.

At the time when the contract is signed, expected profits are:

$$\pi(\mathcal{C}) = \alpha\pi(C_L;\theta_L) + (1-\alpha)\,\pi(C_H;\theta_H)$$

where I evaluate $\pi(C;\theta)$ at the optimal choice of each type of entrepreneur.

The participation constraint for the investor requires that a menu of contracts which offers negative expected profits will never be offered in equilibrium. Thus, if $\mathcal{C}$ is offered in equilibrium, it has to be that

$$\pi(\mathcal{C}) \geq 0 \tag{3.2}$$

The assumption of free-entry then implies that investors make zero-profits in expectation, that is, (3.2) holds with equality.

3.2.3 Equilibrium Definition

I now turn to the definition of equilibrium for this economy. This is the definition of equilibrium for the full model. I will then adapt this definition to the special cases considered in sections 3.3 and 3.4. Also, as explained above, in section 3.8 I will consider an extension of the model and change the definition of equilibrium accordingly.

Definition. *An equilibrium for the economy is a menu of contracts $\mathcal{C}^* \subseteq \mathbb{C}^{MH}$, a distribution for the asset price $\Phi_P : \mathbb{R} \times \Theta \times \varrho \to [0,1]$, and a demand schedule $D_i : \Theta \times \mathbb{R} \times \varrho \to \mathbb{R}$ for each trader $i \in [0,1]$ such that*

1. *Investors choose $\mathcal{C}^* \subseteq \mathbb{C}^{MH}$ to maximize their profits*

$$\mathcal{C}^* \in \arg \max_{\mathcal{C} \subseteq \mathbb{C}^{MH}} \pi(\mathcal{C})$$

2. *After an entrepreneur learns his type, he selects the contract $C_\theta \in \mathcal{C}^*$ to maximize his expected utility*

$$C_\theta \in \arg \max_{C \in \mathcal{C}^*} U(C;\theta)$$

3. *Free-entry among investors drive expected profits to zero*

$$\pi(\mathcal{C}^*) = 0$$

4. *For each value of θ, p, and ρ, each trader chooses $D_i(\theta,p,\rho)$ to maximize expected profits*

$$D_i(\theta,p,\rho) \in \arg\max_x \mathbb{E}\left[\left(\tilde{y}(\theta)\chi_\theta(p,\rho) + l(\theta)(1-\chi_\theta(p,\rho)) - p\right)x|\theta,p,\rho\right] - \frac{x^2}{2} + W$$

5. *For each value of θ and ρ, $\Phi_P(\cdot|\theta,\rho)$ is the distribution of prices generated by the equilibrium in the financial market. That is, for each value of θ, ρ, and noisy traders u, $\Phi_P(\cdot|\theta,\rho)$ is the cdf of the distribution of prices p which solve the equation*

$$p = \mathbb{E}\left[\tilde{y}(\theta)\chi_\theta(p,\rho) + l(\theta)(1-\chi_\theta(p,\rho))\,|\theta,p,\rho\right] - u$$

3.3 Symmetric Information Benchmark

In this section I solve the model under the assumption that both investors and entrepreneurs learn the type of the project after signing the contract. The model with symmetric information is very similar to the model in Holmstrom and Tirole (2000), the only difference is that here there are two different types of entrepreneurs. The symmetric information assumption is useful for two reasons. First, this model provides a benchmark to evaluate the distortions coming from the asymmetry of information. In the next section I show exactly where the contract is distorted and derive the implications of these distortions. Secondly, the model with symmetric information shows in a simple and stark way the trade-off between financing and risk management stressed in Holmstrom and Tirole (2000). The presence of moral hazard does not allow the entrepreneur to pledge as much income as he would like. If an entrepreneur waits for the realization of the liquidity shock, he might be forced to liquidate some positive NPV projects. Thus, the entrepreneur is willing to trade off better insurance – that is, continuation also for higher liquidity shocks – against a lower scale of the project.

Now, consider the problem of an investor who has to decide which menu of contracts to offer. Of course, the investor offers only contracts that satisfy the participation constraint (3.2). It is also quite intuitive that we can restrict attention to menus containing only two contracts. After the investor and the entrepreneur learn the type of the project, the entrepreneur selects the contract in the menu that maximizes his payoff. Since there are only two types of entrepreneurs, having more than two contracts in a menu is redundant.

As already pointed out above, free-entry and competition among investors push their expected profits to zero. Also, as investors in this economy are exactly the same at time 0, they will offer the same menu of contracts to the entrepreneurs. Entrepreneurs, who are also the same at time 0, will choose the same menu. Finally, after learning their type, all the entrepreneurs of type θ will select the same contract in the menu.

The menu of contracts offered in equilibrium maximizes the utility of the entrepreneur subject to the moral hazard constraint and the zero-profit condition for the investors. Formally, the optimal menu $\mathcal{C}^{sym}$ is the solution to the following problem:

$$\max_{\mathcal{C}\subseteq\mathbb{C}^{MH}} U(\mathcal{C}) \tag{P1}$$

subject to

$$\pi(\mathcal{C}) = 0$$

The following Proposition, which is proved in appendix 3.7, characterizes the solution to problem (P1).

Proposition 18. *The solution to problem (P1) is given by a menu of contracts $\mathcal{C}^{sym}$ with the following properties*

1. *The entrepreneur consumes the minimum share of output if the project is successful and nothing otherwise:*

$$\bar{c}^e_\theta(p,\rho) = (R-\rho_0)\,I_\theta,\ \underline{c}^e_\theta(p,\rho) = 0,\ c^e_{\theta,L}(p,\rho) = 0$$

2. *The investor receives all the pleadgeble income if the project is successful and the liquidation value if the project is liquidated:*

$$\bar{c}^i_\theta(p,\rho) = \rho_0 I_\theta,\ \underline{c}^i_\theta(p,\rho) = 0,\ c^i_{\theta,L}(p,\rho) = l(\theta)\,I_\theta$$

3. *For each θ there exists a threshold ρ^*_θ such that $\theta\rho_0 < \rho^*_\theta < \theta R$ and*

$$\chi_\theta(p,\rho) = 1 \text{ if and only if } \rho \leq \rho^*_\theta$$

4. *Initial investment is given by:*

$$I_\theta = \frac{A}{1 - l(\theta) + \int_{\rho^*_\theta}^{\infty}(\rho - \theta\rho_0 + l(\theta))\,dF_\theta(\rho)}$$

From Proposition 18 we can immediately see that in equilibrium both projects are financed. This is a consequence of the assumption that both projects have positive NPV. Without this assumption, the investor would prefer to liquidate the low type project, which he could easily do since information is symmetric.

As already shown in Holmstrom and Tirole (2000), Proposition 18 shows that with symmetric information the entrepreneur wants to pledge as much income as possible. He does so by getting the minimum possible share per unit of investment which satisfies the moral hazard constraint, $R-\rho_0$. Any attempt to compress the entrepreneur's share below this value would violate the moral hazard constraint and would not be feasible.

Another implication of the model is that a project of type θ is continued if and only if the liquidity shock is below a certain value ρ^*_θ which is higher than what would be ex-post optimal for the investor. To see this, imagine that the investor and the entrepreneur do not contract on a particular continuation rule at time 0. In this case, when time 1 comes and a liquidity shock ρ hits the project, the investor will find it optimal to continue if and only if $\rho \leq \theta\rho_0$. That is, the investor will continue whenever the pledgeable part of income is sufficient to cover the reinvestment outlay. Proposition 18 shows that in the optimal solution the entrepreneur prefers a higher insurance given by a threshold $\rho^*_\theta > \theta\rho_0$. This comes at a cost. The expression for I_θ shows that the higher insurance is traded off against the initial invstment.

3.4 Asymmetric Information without financial markets

In this section I reintroduce asymmetric information after the contract is signed. In particular, after an investor and an entrepreneur meet and agree on a menu of contracts, the entrepreneur learns the type of the project, but the investor does not. The entrepreneur has then an informational advantage over the investor which he will use to his benefit. At time 0, the investor has to take this into account when offering the menu of contracts.

Before solving the full model in section 3.5, I consider the case with no asset market. To fix ideas, we can think of a bank writing a contract to a private equity firm. Equivalently, this exercise can be thought of as if lenders were exogenously restricted to writing contracts which are not contingent on asset prices. This special case serves two purposes. First, it represents another useful benchmark for the unrestricted model where investors can offer contracts conditional on market information. Also, this model highlights in a simple way how asymmetric information affects the equilibrium before introducing asset prices. Secondly, to my knowledge, this is the first paper to derive the implications of asymmetric information in a model where agents face a trade-off between financing and liquidity management.

The fact that at the contracting stage investors and entrepreneurs have symmetric information (none of them knows the quality of the project) simplifies the analysis a lot. In this case, in fact, we can model the economy as a Walrasian market in which investors compete by offering fair contracts to the entrepreneurs. From Prescott and Townsend (1984) we know that such an equilibrium always exist and is constrained efficient. Constrained efficiency simplifies the problem since I can find the equilibrium by solving the social planner's problem directly.

By the revelation principle, I can restrict attention to direct mechanisms with truthful revelation. In this enviroment this means that focusing on menus of contracts containing only two contracts, one for each type of entrepreneurs, is without loss of generality. When an entrepreneur learns his type, he will find it optimal to select the contract in the menu corresponding to his type. By doing so, he will reveal his type to the investor. Formally, the equilibrium solves the program

$$\max_{\mathcal{C} \subseteq \mathbb{C}^{MH}} U(\mathcal{C}) \tag{P2}$$

subject to

$$\pi(\mathcal{C}) = 0$$

and to the incentive compatibility (IC) constraints

$$U\left(C_{\theta^i}; \theta^i\right) \geq U\left(C_{\theta^j}; \theta^i\right), \quad i, j = L, H$$

Under our assumptions, it is possible to prove that only the IC constraint for the low type binds in equilibrium. I can then drop the IC contraint for the high type. The following proposition, which is proved in appendix 3.7, describes the solution to problem P2.

Proposition 19. *The solution to problem P2 is given by a menu of contracts $\mathcal{C}^{asym}$ with the following properties:*

1. *The entrepreneur consumes the minimum share of output if the project is successful and nothing otherwise:*

$$c_{\theta}^{e}(\rho) = (R - \rho_0) I_{\theta},\ c_{\theta}^{e}(\rho) = 0,\ c_{\theta,L}^{e}(\rho) = 0$$

2. *The investor receives all the pleadgeble income if the project is successful and the liquidation value if the project is liquidated:*

$$c_{\theta}^{i}(\rho) = \rho_0 I_{\theta},\ c_{\theta}^{i}(\rho) = 0,\ c_{\theta,L}^{i}(\rho) = l(\theta) I_{\theta}$$

3. *For each θ there exists a threshold $\rho_{\theta}^{**}(\theta)$ such that*

$$\chi_{\theta}(\rho) = 1 \text{ if and only if } \rho \leq \rho_{\theta}^{**}$$

where

(a) *the continuation rule for the low type is the same as in the symmetric information benchmark:*

$$\rho_L^{**} = \rho_L^{*}$$

(b) *the continuation rule for the high type is distorted:*

$$\rho_H^{**} < \rho_L^{*}.$$

4. *Initial investment is given by:*

$$I_H = \frac{A}{\left(\begin{array}{c} \alpha\left(1 - l\left(\theta^H\right) + \int_{\rho_H^{**}}^{\infty} \left(\rho - \theta^H \rho_0 + l\left(\theta^H\right)\right) dF_H(\rho)\right) \\ + (1-\alpha)\frac{1-F_L\left(\rho_H^{**}\right)}{1-F_L\left(\rho_L^{**}\right)}\left(1 - l\left(\theta^L\right) + \int_{\rho_L^{**}}^{\infty} \left(\rho - \theta^L \rho_0 + l\left(\theta^L\right)\right) dF_L(\rho)\right) \end{array}\right)}$$

and

$$I_L = \frac{1 - F_L\left(\rho_H^{**}\right)}{1 - F_L\left(\rho_L^{**}\right)} I_H$$

Results (1) and (2) in Proposition 19 show that the payments made to investors and entreprenuers have the same form as those in Proposition 18. As it was the case when information was symmetric, the entrepreneur finds it optimal to pleadge the highest possible share of period 2 return and keep for himself the minimum share of return that satisfies the moral hazard constraint.

Therefore, asymmetric information does not distort the way the optimal contract divides the total surplus between investors and entrepreneurs.

The contract under asymmetric information is, however, distorted. The optimal continuation rule under asymmetric information is still characterized by a threshold value for ρ above which the project is liquidated. However, while the threshold for the low type is identical to the corresponding threshold in the case of symmetric information, the continuation rule for the high type is different. In particular, to ensure that the low type wants to separate from the high type, the contract has to commit to liquidate a higher fraction of high type projects.

The case considered in thie section is not only a useful benchmark, but allows me to draw some independent conclusions. In particular, proposition 19 makes predictions on how the trade-off between financing and liquidity management differs accross projects of different, unknown quality. Thus, the model offers a different explanation on why more productive firms engage in less liquidity management (or less risk management). When firms with different productivities are simultaneously active in the market, the model implies that the more productive firms will separate from the other, less productive firms by giving up some liquidity insurance in exchange for a higher initial investment. This is similar to the conclusion reached in Rampini and Viswanathan (2010) in a model where firms are subject to endogenous borrowing constraints. There is, however, an important difference. In Rampini and Viswanathan (2010) the more productive firms are those that operate with less capital. With a concave production function, in fact, a smaller firm is at the margin more productive than a bigger firm. Things are different in my model. First, I assume constant returns to scale, so the productivity of a firm is independent of its scale of production. More importantly, I am assuming that the productivity of a firm, its type, is unobservable. If productivity was determined uniquely by the capital of the firm, which is arguably an observable characteristic, then the asymmetric information problem would disappear. My model, therefore, shows that even firms with the same characteristic, but with different projects of unknown qualities, can make different choices of financing and liquidity management.

The model with asymmetric information has another implication. We know that high type entrepreneurs are continued less often than they would be if information was symmetric. But what happens to initial investment? If we look at the expression for the optimal choice of I_L, we can see that the initial investment of the low type is the same as in the symmetric information case. The interesting action comes from I_H. As observed before, in the optimal contract the high type chooses to give up some insurance in exchange for a higher initial investment. The interesting thing, however, is that there is no general implication on how I_H compares to its symmetric information counterpart. Indeed, it could well be the case that the investment in the high project under asymmetric information is higher than the investment that would occur under symmetric information.

To gain some intuition for this results, remember the fundamental trade-off between initial financing and liquidity insurance in the Holmstrom-Tirole model presented in section 3.3. There, the investor and the entrepreneur signed a contract where the investor was committed to continue the project for higher values of ρ in exchange for a lower initial investment. Now, Proposition 19 states that, under asymmetric information, the high type project is continued only for lower values of ρ. Thus, the comparison between initial investment under symmetric information and initial investment under asymmetric information breaks down to the comparison between the new threshold ρ_H^{**} and the ex-post optimal continuation rule for the investor, $\theta^H \rho_0$. More specifically, if $\rho_H^{**} \geq \theta^H \rho_0$, asymmetric information reduces the scope for liquidity insurance, but at the same time makes the project *more* profitable for the investors who are happy to make a higher initial investment. On the contrary, if asymmetric information calls for a contract so distorted that $\rho_H^{**} < \theta^H \rho_0$, then the high type project will be less profitable for both the investors and the entreprenuers. In this case, I_H is lower than its symmetric information counterpart.

3.5 Asymmetric Information with financial markets

Let's now turn to the full model where the price of the asset in verifiable and can be contracted upon.

Remember that the price is informative about the type of the project, but it is observed only *after* the contract is signed. Since in equilibrium there is truthful revelation, and the entrepreneur reports his type *before* the price is observed, it is not clear why the price should enter the optimal contract.

In this paper, the principal is the investor, the agent is the entrepreneur, and the ex-post signal is the price of the asset. The important difference is that the signal of my model is endogenously determined in equilibrium together with all the other elements of the contract. In particular, the liquidation rule specified in the contract affects the distribution of the price.

The possibility of observing an endogenous price that signals the type of the agent raises the issue of whether the investor is sophisticated enough to take into account how the distribution of the price is affected by the contract. An investor who fails to take this into account when signing the contract can lose a substantial amount of money. In what follows I assume that investors fully understand how the contract they offer affects the equilibrium distribution of the asset price.

Let me change the problem to allow investors and entrepreneurs to condition the optimal contract on the asset price. Remember that $\Phi_p(\cdot|\theta,\rho)$ is the cdf of the price distribution which is determined from the equilibrium in the financial market. The equilibrium is the solution to:

$$\max_{\mathcal{C}\subseteq\mathcal{C}^{MH}} U\left(\mathcal{C};\Phi_p\left(\cdot|\theta,\rho\right)\right) \tag{P3}$$

subject to the contract delivering zero profits to the investor who offers it:

$$\pi\left(\mathcal{C};\Phi_p\left(\cdot|\theta,\rho\right)\right)=0$$

and the incentive compatibility constraint for the two types:

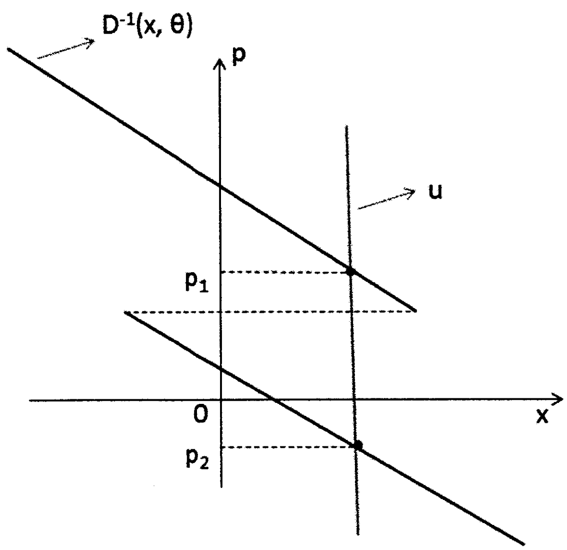
$$U\left(C_{\theta^i};\theta^i,\Phi_p\left(\cdot|\theta^i,\rho\right)\right)\geq U\left(C_{\theta^j};\theta^i,\Phi_p\left(\cdot|\theta^i,\rho\right)\right),\quad i,j=L,H$$

To solve (P3) I first need to find the equilibrium in the financial market. Remember from definition 3.2.1 that the equilibrium asset price is given by:

$$p=\mathbb{E}\left[\tilde{y}\left(\theta\right)\chi_\theta\left(p,\rho\right)+l\left(\theta\right)\left(1-\chi_\theta\left(p,\rho\right)\right)|\theta,p,\rho\right]-u \tag{3.3}$$

Consider the continuation rule for type θ, $\chi_\theta(p,\rho)$. If for the pair (p,ρ) the project is continued, the expected return is $\mathbb{E}\left[\tilde{y}\left(\theta\right)|\theta\right]=R\theta$, otherwise, the return is $l\left(\theta\right)$. The quadratic objective of the traders then implies that the demand for the asset is equal to $R\theta-p$ if the project is continued, and is equal to $l\left(\theta\right)-p$ if the project is liquidated. The equilibruium is given by the price where this demand intersects the random supply u.

The following graph plots the inverse demand and the supply of the asset for a project of type θ and a liquidity shock ρ.



The graph highlights an important point. The dependence of the asset price on the continuation rule $\chi_\theta(p,\rho)$ will in general lead to multiple equilibria for certain values of the supply u. In the region where multiple equilibria exist, the project will be continued if the equilibrium with the higher price is selected and will be terminated otherwise.

When multiple equilibria exist the equilibrium distribution of the asset price depends on the equilibrium selection criterion. In general, a selection criterion could be a function of other objects of the model like, for example, the liquidity shock or the specific realization of the random supply. Also, a selection criterion could also depend on some parameters of the model. This would make comparative static exercises particularly hard to perform. For simplicity, I am simply going to assume these issues away and consider a random selection criterion which is independent of the other shocks or parameters of the model. More specifically, I am going to assume that whenever there are two equilibrium prices, the higher price is selected with probability γ.

The investor and the entrepreneur know the specific form of the selection criterion and take this into account when signing the contract. In appendix 3.7.2 I show that, if the parameters of the model satisfy the condition (3.6), then there exists a threshold $\bar{p}_\theta(\rho)$ such that $\chi_\theta(p,\rho)$ can be represented as:

$$\chi_\theta(p,\rho) = \begin{cases} 1 & \text{if } p \geq \bar{p}_\theta(\rho) \\ 0 & \text{otherwise} \end{cases} \tag{3.4}$$

The assumption that is needed to prove this result is essentially a requirement that if an investor wants to continue a project for a given p, then he wants to continue the project also for higher realizations of the asset price. The reason why this restriction is not automatically satisfied has to do with the possibility of multiple equilibria in the financial market.

With this representation of $\chi_\theta(p,\rho)$, it is easy to derive the probability density function of the equilibrium distribution of the asset price:

$$\phi_P(p|\theta,\rho) = \begin{cases} \phi(R\theta - p) & \text{if } p \geq R\theta + \bar{p}_\theta(\rho) - l(\theta) \\ \gamma\phi(R\theta^L - p) & \text{if } \bar{p}_\theta(\rho) \leq p < R\theta + \bar{p}_\theta(\rho) - l(\theta) \\ (1-\gamma)\phi(l(\theta) - p) & \text{if } \bar{p}_\theta(\rho) - R\theta + l(\theta) \leq p < \bar{p}_\theta(\rho) \\ \phi(l(\theta) - p) & \text{if } p < \bar{p}_\theta(\rho) - R\theta + l(\theta) \end{cases} \tag{3.5}$$

This distribution comes directly from (3.3) under our assumption that, for those values of p for which there are multiple equilibria, the higher price equilibrium is selected with probability γ. The interpretation for this price distribution is simple. The probability of observing a particular price p is given by the distribution of the noise ϕ evaluated at the value of u given by the equilibrium in the financial market. Also, if an investor observes a price p which is in the range where multiple equilibria exist, he will have to take into account the selection criterion. This is the reason why γ appears in (3.5) when $p \in [\bar{p}_\theta(\rho) - R\theta + l(\theta), R\theta + \bar{p}_\theta(\rho) - l(\theta)]$.

Optimal continuation rules. The next step is to substitute the equilibrium price distribution (3.5) into P3 and solve for the optimal contract. In appendix 3.7.2, I show that it is possible to find the optimal continuation rules $\chi_\theta(p,\rho)$ in P3 by maximizing the objective function pointwise for each value of (p,ρ). This leads to a fixed point problem which I solve numerically. More specifically, in appendix 3.7.2, I show that the optimal continuation rule for the low type does not depend on realization of the asset price, that is, $\chi_L(p,\rho) = \chi_L(\rho)$, $\forall p$. This is intuitive: remember that the asset price is useful for the investor only insofar as it gives the entrepreneur better incentives to truthfully reveal his type. However, under my assumptions, only the low type has an incentive to misreport his type. The investor and the entrepreneur find it optimal to condition the contract on the price and, therefore, to let the continuation decision partly depend on the extra noise in the contract (that is, the noise due to the random supply and the equilibrium selection when multiple equilibria exist) only if this can help relax the ex-ante problem. Since only the incentive constraint of the low type binds in equilibrium, there is no reason to have the optimal contract for the low type depend on the asset price.

Formally, there exists a threshold $\bar{\rho}_L$ such that $\bar{p}_L(\rho) = -\infty$ for $\rho \leq \bar{\rho}_L$ and $\bar{p}_L(\rho) = \infty$ for $\rho > \bar{\rho}_L$. Hence, the continuation rule (3.4) for the low type becomes:

$$\chi_L(\rho) = \begin{cases} 1 & \text{if } \rho \leq \bar{\rho}_L \\ 0 & \text{otherwise} \end{cases}$$

Interestingly, it is possible to prove that $\bar{\rho}_L$ coincides with the symmetric information threshold in section 3.3 which, by Proposition 19, is also the same as the asymmetric information threshold without asset prices. Again, this is quite intuitive: distorting the liquidity insurance for the low type would only harm the investor since the low type would have a greater incentive to misreport his type.

On the other hand, the optimal continuation rule for the high type $\chi_H(p,\rho)$ depends on the asset price. Formally, let V^* be the value of P3, that is, the expected utility of the entrepreneur evaluated at the optimal contract. Also, let V_L^* be the expected utility that a low type entrepreneur would get if the investor knew his type *before* signing the contract. The following lemma, which I prove in appendix 3.7.2, provides a simple characterization of the optimal threshold $\bar{p}_H(\rho)$.

Lemma 8. *The optimal threshold for the high type $\bar{p}_H(\rho)$ in (3.4) is the unique solution to:*

$$\frac{g^L(\bar{p}_H(\rho))\, f_L(\rho)}{g^H(\bar{p}_H(\rho))\, f_H(\rho)} = \frac{\alpha}{1-\alpha} \frac{\left(\theta^H R - l(\theta^H)\right) - V^*\left(\rho - \theta^H \rho_0 + l(\theta^H)\right)}{\left(\theta^L R - l(\theta^L)\right)\left(\frac{V^*}{V_L^*} - 1\right)} \tag{3.6}$$

where

$$g^i(\bar{p}_H(\rho)) \equiv \gamma\phi\left(\left(R\theta - \bar{p}_H(\rho)\right)/\sigma_u\right) + (1-\gamma)\,\phi\left(\left(l(\theta) - \bar{p}_H(\rho)\right)/\sigma_u\right), \quad i = L, H$$

is the pdf of a mixture of two Normal random variables with mean $R\theta$ and $l(\theta)$, respectively, evaluated at the optimal threshold $\bar{p}_H(\rho)$.

The left-hand side of (3.6) is the likelihood ratio of the pair $(\bar{p}_H(\rho), \rho)$ when $g^i(\bar{p}_H(\rho))$ is used as pdf for $\bar{p}_H(\rho)$. From Lemma 8 we can see that $g^i(\bar{p}_H(\rho))$ is a mixture of two Normal random variables whenever $\gamma \in (0,1)$. This comes from my assumptions on the equilibrium selection criterion.

While this problem has to be solved numerically, for given values of V^* and V_L^* I can use equation (3.6) to characterize the optimal threshold $\bar{p}_H(\rho)$ separately for each ρ – optimal thresholds corresponding to different values of ρ are connected only through the values of V^* and V_L^*. I can then prove some properties of $\bar{p}_H(\rho)$ without having to solve the overall problem (that is, without having to compute V^* and V_L^*).

The optimal value of the problem V^* is greater than the optimal value V_L^*, hence the denominator of the right-hand side of (3.6) is positive. Thus, if ρ is high enough that the right-hand side is negative, that is, if

$$\rho > \frac{\theta^H R - l\left(\theta^H\right)}{V^*} + \theta^H \rho_0 - l\left(\theta^H\right)$$

then the optimal threshold will be ∞. That is, for high enough values of ρ, the project will be liquidated no matter what the price is.

Also, the ratio $g^L\left(\bar{p}_H(\rho)\right)/g^H\left(\bar{p}_H(\rho)\right)$ is the contribution of the asset price to the optimal continuation rule. Under the assumption that $l\left(\theta^H\right) \neq l\left(\theta^L\right)$, the asset price is informative both when the firm is continued and when the firm is liquidated. The ratio $g^L\left(\bar{p}_H(\rho)\right)/g^H\left(\bar{p}_H(\rho)\right)$ is, in fact, given by combining these two cases with weight γ.

The fact that (3.6) depends on V^*, which itself depends on $\chi_H(p,\rho)$[3] implies that solving this equation for $\bar{p}_H(\rho)$ is not enough to characterize the optimal continuation rule. However, since for each ρ the optimal threshold $\bar{p}_H(\rho)$ depends on all the other thresholds $\bar{p}_H(\rho')$, $\rho' \neq \rho$, only through V^*, I can solve P3 by solving a fixed point in V^*. In particular, given an initial value for V^*, call it V_0^*, I can solve (3.6) to get the thresholds $\{\bar{p}_H(\rho; V_0^*)\}$ (I have made explicit the dependence on V_0^*) and use them to compute the new value of P3, call it V_1^*. The fixed point of this algorithm is the optimal value V^* and, therefore, the optimal thresholds are $\bar{p}_H(\rho; V^*) \equiv \bar{p}_H(\rho)$. In appendix 3.7 I define this fixed point problem more formally.

The next proposition summarizes these results.

Proposition 20. *The solution to P3 is given by a menu of contracts C^{asymp} with the following properties:*

1. *The entrepreneur consumes the minimum share of output if the project is successful and nothing otherwise:*

$$\bar{c}^e_\theta(p,\rho) = (R-\rho_0)\, I_\theta,\ \underline{c}^e_\theta(p,\rho) = 0,\ c^e_{\theta,L}(p,\rho) = 0$$

2. *The investor receives all the pleadgeble income if the project is successful and the liquidation value if the project is liquidated:*

$$\bar{c}^i_\theta(p,\rho) = \rho_0 I_\theta,\ \underline{c}^i_\theta(p,\rho) = 0,\ c^i_{\theta,L}(p,\rho) = l(\theta)\, I_\theta$$

[3]The dependence on V_L^* is not an issue here since V_L^* depends only on $\chi_{\theta^L}(\rho)$, which can be computed separately from $\chi_{\theta^H}(p,\rho)$.

3. The low type obtains the same liquidity insurance as in the symmetric information benchmark:

$$\chi_L(\rho) = \begin{cases} 1 & \text{if } \rho \leq \bar{\rho}_L \\ 0 & \text{otherwise} \end{cases}$$

4. The continuation rule for the high type $\chi_H(p, \rho)$ depends on the asset price. Formally, for each ρ there exists a threshold $\bar{p}_H(\rho)$ such that

$$\chi_H(p, \rho) = \begin{cases} 1 & \text{if } p \geq \bar{p}_H(\rho) \\ 0 & \text{otherwise} \end{cases}$$

where $\bar{p}_H(\rho)$ is the unique solution to (3.6).

5. The initial investment levels are given by (3.15) and (3.16) in appendix 8.

6. The optimal contract determines an equilibrium distribution for the asset price $\Phi_P(p|\theta, \rho)$ given by (3.5).

3.5.1 Comparative Statics

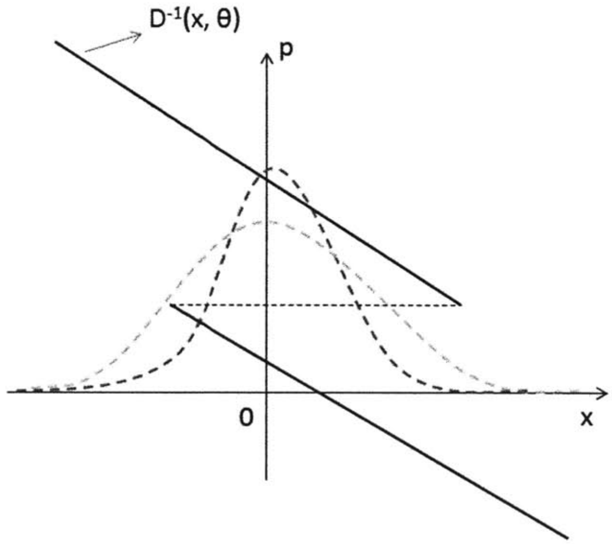
Now that I have characterized the equilibrium, I can answer the interesting question of what would happen if financial markets were to become less informative about firms' fundamentals. In my model, a less informative financial market can be captured by increasing the noise of the random supply σ_u^2. A noisier supply makes it harder for investors to learn the fundamental of a firm from its asset price.

More specifically, I consider the effect on the optimal contract of a change of σ_u^2, both in the case that this change is anticipated by the agents when the contract is signed, and in the case that it is not. Of course, in the former case the change of σ_u^2 is going to be incorporated in the contract. On the other hand, a non-anticipated increase of σ_u^2, can be thought of as a realization of a zero-probability event after investment and liquidity management decisions are made.

While the case that the change of σ_u^2 is totally unexpected might sound extreme, it may nonetheless capture some of the events happening during the recent financial crisis. To be sure, financial market crashes are not totally uncommon and we can expect investors and entrepreneurs to somehow take this risk into account when deciding on liquidity management. It is also true, however, that the severity of this crisis stroke many market participants, at least in part, as completely unexpected.

The reason why financial crises are associated with less informative asset prices comes from the idea that the informed traders are also those with higher leverage. Think of informed traders as investments banks or hedge funds. When prices drop, high leverage investors are hit more severily than other investors and are forced to withdraw from the market (either because they go bankrupt or simply because they are subject to margin calls). With fewer and more constrained informed investors, financial markets become less informative. This mechanism has been studied recently by Angeletos and Lorenzoni (2010). Here, I am going to take a much more reduced form route and simply consider exogenous variations of σ_u^2. It is important to keep in mind, however, what is the rationale behind this comparative static exercise.

Remember that σ_u^2 enters the problem through (3.5). In particular, for given contract and hence for given threshold $\bar{p}_H(\rho)$, an increase of σ_u^2 makes the equilibrium distribution $\Phi_P(p|\theta,\rho)$ more dispersed. Of course, as I said above, the overall effect of a change of σ_u^2 depends on whether this change was anticipated or not and, therefore, on whether $\bar{p}_H(\rho)$ changes or not. Let's first analize the case where the change is not anticipated. By assumption, the contract signed at time 0 does not incorporate the change of σ_u^2 and, in particular, the optimal threshold $\bar{p}_H(\rho)$ is exactly the same as before. When time 1 arrives, investors and entrepreneurs face a new and unexpected price distribution $\Phi_P'(p|\theta,\rho)$.



A noisier supply implies that in equilibrium more extreme realizations of asset prices are observed with higher probability. For given threshold $\bar{p}_H(\rho)$, a higher price implies that the firm is continued more often, while a lower price implies the opposite. After an unexpected increase of σ_u^2, therefore, liquidation of good projects can increase or decrease depending on the shape of the demand function.

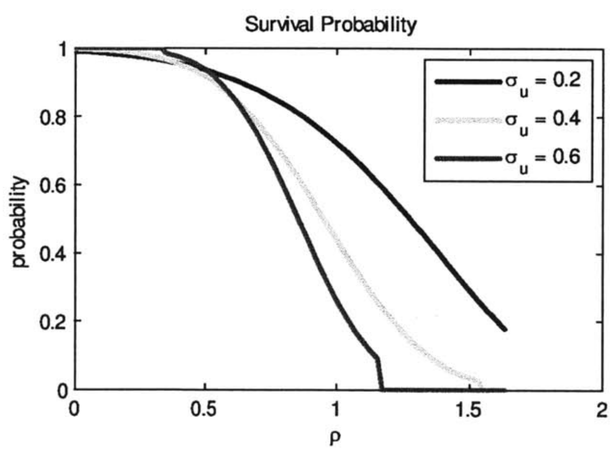
Proposition 21. *At the optimal contract C^{asymp}, for each ρ the probability of continuation is given by*

$$\Pr\left(p \geq \bar{p}_H(\rho)\,|\rho\right) = \gamma\Phi\left(R\theta^H - \bar{p}_H(\rho)\right) + (1-\gamma)\,\Phi\left(l\left(\theta^H\right) - \bar{p}_H(\rho)\right)$$

Furthermore, there exists a threshold $\tilde{p}_H(\gamma) > 0$ such that $\Pr\left(p \geq \bar{p}_H(\rho)\,|\rho\right)$ is decreasing in σ_u^2 for $\bar{p}_H(\rho) \leq \tilde{p}_H(\gamma)$ and it is increasing in σ_u^2 for $\bar{p}_H(\rho) > \tilde{p}_H(\gamma)$.
Finally, $d\tilde{p}_H(\gamma)/d\gamma < 0$.

Proposition 21 shows that the effect of σ_u^2 on the probability that a firm is liquidated depends on the threshold $\bar{p}_H(\rho)$ and hence on the liquidity shock ρ. Firms that experience low liquidity shocks and, therefore, are continued even if asset prices are not so high (i.e., $\bar{p}_H(\rho)$ is low) will face a lower probability of being continued. This is true because when σ_u^2 increases the probability of observing more extreme realizations of the asset price increases. The opposite is true for high values of ρ.

The alternative case is when investors and entrepreneurs expect financial markets to become less informative and incorporate this into the contract. It is easy to imagine how the result will look like. As the asset supply becomes noisier, investors will rely less on the market. In the limit, the market becomes completely uninformative and the model collapses into the special case of section 3.4. To confirm this intuition let's reconsider equation (3.6). The right hand side of that equation does not depend on σ_u^2, while the left hand side depends on σ_u^2 through the likelihood ratio $g^L(\bar{p}_H(\rho))/g^H(\bar{p}_H(\rho))$. Now, the limit of $g^L(\bar{p}_H(\rho))/g^H(\bar{p}_H(\rho))$ as σ_u^2 grows to infinity is 1. This means that in the limit as the asset price becomes completely uninformative, investors stop using it and (3.6) becomes the same as the condition that defined $\rho_{\theta H}^{**}$ in Proposition 19. The following graph plots the survival probability – that is, $\Pr(p \geq \bar{p}_H(\rho)|\rho)$ – as a function of ρ for for three different values of σ_u^2. In the simulations I have assumed $l(\theta^H) = l(\theta^L) = 0$ and $\gamma = 1$.



We already know that the probability that a firm is continued decreases when higher liquidity shocks hit the firm. More to the point, the figure shows that as σ_u^2 increases, the graph of $\Pr(p \geq \bar{p}_H(\rho)|\rho)$ tends to a vertical line. In other words, $\Pr(p \geq \bar{p}_H(\rho)|\rho)$ converges to a step function that is equal to 1 or 0 depending on the value of ρ. This is exactly how the survival probability would look like were asset prices not available to investors.

3.6 Conclusion

The model of this paper shows how liquidity management and investment decisions are affeced by the presence of asymmetric information between investors and entrepreneurs. With asymmetric information more productive firms give up some liquidity insurance to separate from less productive firms and obtain a higher initial investment. Relative to the symmetric information case, the distorted liquidity insurance, while constrained efficient, causes a higher share of good projects to be liquidated.

Moreover, market signals – such as the price of a security that is correlated with the fundamental of the firm – can mitigate the asymmetric information problem. I show that asset prices can be useful even if on equilibrium investors can perfectly screen the different types of borrowers. Indeed, financial markets are used by the investors only insofar as they help screen the borrowers more efficiently. On equilibrium, investors are willing to accept that the extra noise in asset prices partly determines the fate of valuable firms in order to screen the different types of entrepreneurs more efficiently.

The idea of using market signals to relax the incentive constraint of the investors is novel to this literature and has some interesting implications. For example, the amount of insurance that firms can get in equilibrium depends on the ability of financial markets to convey information about fundamentals. Thus, if during a financial crisis informed traders lose wealth and prices become less informative, then this model shows another possible mechanism through which market disruptions can lead many valuable firms to bankruptcy. The endogeneity of asset prices, however, may hinder the extent to which it is possible to use financial markets. In particular, I show that when liquidation decisions are based on asset prices – as they are in the optimal contract – multiple equilibria in financial markets may be possible.

The model focuses on a partial equilibrium contracting problem. In particular, while focusing on the demand of liquidity of entrepreneurs, the model has ignored how liquidity is supplied at time 1. A possible extension would be to let the the supply of funds in period 1 depend on the investment choices made in period 0. More specifically, I could imagine investors deciding how much money to invest in the project at time 0 and how much cash to carry to time 1 in order to cover the liquidity shocks. At time 1, investors with an excess of funds would lend money to the investors who are short of funds. Bhattacharya and Gale (1987) address this issue and show that investors do not take into account how their portfolio decision at time 0 impacts the overall availability of funds in period 1. This results is an inefficiently low amount of funds at time 1 (see also Jacklin (1987) and Allen and Gale (1997)). The model of this paper could be extended to incorporate an endogenous supply of funds at time 1, at least conceptually, in a relatevely easy way.

3.7 Appendix 1. Proofs

3.7.1 Proof of results (1) and (2) in Propositions 18, 19, and 20

Denote with S the expected surplus originated by the project, that is

$$\begin{aligned} S\left(\chi_{\theta^H},\chi_{\theta^L},I_H,I_L\right) &\equiv \alpha\theta^H R I_H \int\int \chi_{\theta^H}(p,\rho)\, d\Phi_P\left(p|\theta^H,\rho\right) dF_{\theta^H}(\rho) \\ &+ (1-\alpha)\,\theta^L R I_L \int\int \chi_{\theta^L}(p,\rho)\, d\Phi_P\left(p|\theta^L,\rho\right) dF_{\theta^L}(\rho) \\ &+ \alpha l\left(\theta^H\right) I_H \int\int \left(1-\chi_{\theta^H}(p,\rho)\right) d\Phi_P\left(p|\theta^H,\rho\right) dF_{\theta^H}(\rho) \\ &+ (1-\alpha)\, l\left(\theta^L\right) I_L \int\int \left(1-\chi_{\theta^L}(p,\rho)\right) d\Phi_P\left(p|\theta^L,\rho\right) dF_{\theta^L}(\rho) \end{aligned}$$

and with $\mathbb{E}^{\theta}\left[\mathbf{c}^e_{\theta'}\chi_{\theta'}\right]$ the expected payment to the entrepreneur of type θ if he reports type θ', that is,

$$\begin{aligned} \mathbb{E}^{\theta}\left[\mathbf{c}^e_{\theta'}\chi_{\theta'}\right] &= \theta\int\int \overline{c}^e_{\theta}(p,\rho)\,\chi_{\theta}(p,\rho)\, d\Phi_P\left(p|\theta',\rho\right) dF_{\theta'}(\rho) \\ &+ (1-\theta)\int\int \underline{c}^e_{\theta}(p,\rho)\,\chi_{\theta}(p,\rho)\, d\Phi_P\left(p|\theta',\rho\right) dF_{\theta'}(\rho) \\ &+ \int c^e_{\theta,L}(p,\rho)\left(1-\chi_{\theta}(p,\rho)\right) d\Phi_P\left(p|\theta',\rho\right) dF_{\theta'}(\rho) \end{aligned}$$

The problem is then

$$\max_{\left\{\mathbf{c}^e,\chi_{\theta^H},\chi_{\theta^L},I_H,I_L\right\}} S\left(\chi_{\theta^H},\chi_{\theta^L},I_H,I_L\right)$$

subject to

$$\begin{aligned} & S\left(\chi_{\theta^H},\chi_{\theta^L},I_H,I_L\right) - \alpha\mathbb{E}^{\theta^H}\left[\mathbf{c}^e_{\theta^H}\chi_{\theta^H}\right] - (1-\alpha)\,\mathbb{E}^{\theta^L}\left[\mathbf{c}^e_{\theta^L}\chi_{\theta^L}\right] - \alpha I_H - (1-\alpha)\, I_L + A \\ - \quad & \alpha I_H\int\int \rho\chi_{\theta^H}(p,\rho)\, d\Phi_P\left(p|\theta^H,\rho\right) dF_{\theta^H}(\rho) - (1-\alpha)\, I_L\int\int \rho\chi_{\theta^L}(p,\rho)\, d\Phi_P\left(p|\theta^L,\rho\right) dF_{\theta^L}(\rho) \geq 0 \end{aligned} \tag{3.7}$$

$$\mathbb{E}^{\theta^L}\left[\mathbf{c}^e_{\theta^L}\chi_{\theta^L}\right] \geq \mathbb{E}^{\theta^L}\left[\mathbf{c}^e_{\theta^H}\chi_{\theta^H}\right] \tag{3.8}$$

$$\mathbb{E}^{\theta^H}\left[\mathbf{c}^e_{\theta^H}\chi_{\theta^H}\right] \geq \mathbb{E}^{\theta^H}\left[\mathbf{c}^e_{\theta^L}\chi_{\theta^L}\right] \tag{3.9}$$

$$\overline{c}^e_{\theta^H}(p,\rho) \geq (R-\rho_0)\, I_H, \quad \overline{c}^e_{\theta^L}(p,\rho) \geq (R-\rho_0)\, I_L \tag{3.10}$$

$$\begin{aligned} \underline{c}^e_{\theta^H}(p,\rho) &\geq 0, \quad \underline{c}^e_{\theta^L}(p,\rho) \geq 0 \\ c^e_{\theta^H,L}(p,\rho) &\geq 0, \quad c^e_{\theta^L,L}(p,\rho) \geq 0 \end{aligned} \tag{3.11}$$

In words, the contract has to maximize the total surplus subject to the investors making non-negative profits (3.7), the two incentice compatibility constraints for the entrepreneurs, (3.9) and (3.8), the moral hazard constraints for the entrepreneurs (3.10), and the limited liability constraint for the entrepreneur (3.11).

Under my assumption the low type will want to pretend to be high type, but not vice versa. Thus, in the maximization problem I will ignore the IC constraint for the high type.

Let μ, ν, $\theta^H \gamma_H(p,\rho)\,\phi_P(p|\theta^H,\rho)\,f_{\theta^H}(\rho)$, $\theta^L \gamma_L(p,\rho)\,\phi_P(p|\theta^L,\rho)\,f_{\theta^L}(\rho)$, $(1-\theta^H)\,\varphi_H(p,\rho)\,\phi_P(p|\theta^H,\rho)\,f_{\theta^H}(\rho)$, $(1-\theta^L)\,\varphi_L(p,\rho)\,\phi_P(p|\theta^L,\rho)\,f_{\theta^L}(\rho)$ be the Lagrange multipliers on the constraints, respectively. The first order condition w.r.t. $\overline{c}^e_{\theta^H}(p,\rho)$, $\underline{c}^e_{\theta^H}(p,\rho)$, $\overline{c}^e_{\theta^L}(p,\rho)$, and $\underline{c}^e_{\theta^L}(p,\rho)$ are, respectively,

$$-\mu\alpha\chi_{\theta^H}(p,\rho) - \nu\frac{\theta^L}{\theta^H}\chi_{\theta^H}(p,\rho) + \gamma_H(p,\rho) \geq 0$$

$$-\mu\alpha\chi_{\theta^H}(p,\rho) - \nu\frac{1-\theta^L}{1-\theta^H}\chi_{\theta^H}(p,\rho) + \varphi_H(p,\rho) \geq 0$$

$$-\mu(1-\alpha)\chi_{\theta^L}(p,\rho) + \nu\chi_{\theta^L}(p,\rho) + \gamma_L(p,\rho) \geq 0$$

$$-\mu(1-\alpha)\chi_{\theta^L}(p,\rho) + \nu\chi_{\theta^L}(p,\rho) + \varphi_L(p,\rho) \geq 0$$

$$-\mu\alpha\left(1-\chi_{\theta^H}(p,\rho)\right) - \nu\frac{\theta^L}{\theta^H}\left(1-\chi_{\theta^H}(p,\rho)\right) + \delta_H(p,\rho) \geq 0$$

$$-\mu(1-\alpha)\left(1-\chi_{\theta^L}(p,\rho)\right) + \nu\left(1-\chi_{\theta^L}(p,\rho)\right) + \delta_L(p,\rho) \geq 0$$

Note that if the constraint $\overline{c}^e_{\theta^H}(p,\rho) \geq (R-\rho_0)\,I_H$ and the constraint $\underline{c}^e_{\theta^H}(p,\rho) \geq 0$ didn't bind for a pair (p,ρ) for which the firm is continued (i.e., $\chi_{\theta^H}(p,\rho) = 1$), then $\gamma_H(p,\rho) = 0$ and $\varphi_H(p,\rho) = 0$. This would make the first two conditions negative and contradict the fact that $\overline{c}^e_{\theta^H}(p,\rho) \geq (R-\rho_0)\,I_H$ and $\underline{c}^e_{\theta^H}(p,\rho) \geq 0$ didn't bind.

Therefore, it is optimal to set

$$\overline{c}^e_{\theta^H}(p,\rho) = (R-\rho_0)\,I_H$$

and

$$\underline{c}^e_{\theta^H}(p,\rho) = 0$$

Also, we must have

$$-\mu(1-\alpha)\,R\chi_{\theta^L}(p,\rho) + \nu R\chi_{\theta^L}(p,\rho) + \gamma_L(p,\rho) \leq 0$$

$$-\mu(1-\alpha)\,R\chi_{\theta^L}(p,\rho) + \nu R\chi_{\theta^L}(p,\rho) + \varphi_L(p,\rho) \leq 0$$

Otherwise, we would have either $\overline{c}^e_{\theta^L}(p,\rho) = \infty$ or $\underline{c}^e_{\theta^L}(p,\rho) = \infty$ which violate (3.7).

I am going to show that also for the low types the moral hazard and limited liability constraints must bind.

If the two conditions above hold with equality for a pair (p, ρ) for which $\chi_{\theta^L}(p, \rho) = 1$, then we can choose $\bar{c}^e_{\theta^L}(p, \rho)$ and $\underline{c}^e_{\theta^L}(p, \rho)$ so that the constraints $\bar{c}^e_{\theta^L}(p, \rho) \geq (R - \rho_0) I_L$ and $\underline{c}^e_{\theta^L}(p, \rho) \geq 0$ don't bind. In turn, this implies that $\gamma_L(p, \rho) = 0$ and $\varphi_L(p, \rho) = 0$ and both conditions become:

$$-\mu(1 - \alpha) + \nu = 0$$

But if this equality is true, then $\gamma_L(p, \rho) = 0$ and $\varphi_L(p, \rho) = 0$ for all (p, ρ) for which $\chi_{\theta^L}(p, \rho) = 1$. To see this, observe that if for some pair (p, ρ) we had $\gamma_L(p, \rho) > 0$ or $\varphi_L(p, \rho) > 0$, then either $\bar{c}^e_{\theta^L}(p, \rho)$ or $\underline{c}^e_{\theta^L}(p, \rho)$ would be infinite.

Now, assume that $-\mu(1 - \alpha) + \nu = 0$, and consider a pair (p, ρ) for which $\chi_{\theta^L}(p, \rho) = 0$. Suppose that either $\gamma_L(p, \rho) > 0$ (the moral hazard constraint is binding) or $\varphi_L(p, \rho) > 0$ (the limited liability constraint is binding). The former would imply $\bar{c}^e_{\theta^L}(p, \rho) = \infty$, and the latter $\underline{c}^e_{\theta^L}(p, \rho) = \infty$. Both of these values contradict the fact that, by assumption, either the moral hazard or the limited liability constraint is binding. Therefore, it must be true that $\gamma_L(p, \rho) = 0$ and $\varphi_L(p, \rho) = 0$ for every pair (p, ρ) such that $\chi_{\theta^L}(p, \rho) = 0$. I have already proved that $\gamma_L(p, \rho) = 0$ for every pair (p, ρ) such that $\chi_{\theta^L}(p, \rho) = 1$. Thus, $\gamma_L(p, \rho) = 0$ for all pairs (p, ρ). This, however, implies that the moral hazard constraint

$$\int \gamma_L(p, \rho) \left(\bar{c}^e_{\theta^L}(p, \rho) - (R - \rho_0) I_L\right) = 0$$

is satisfied for every I_L and, therefore, that $I_L = \infty$. The latter violates the (3.7) constraint.

Thus, the optimal payments to the entreprenuer must be such that

$$\bar{c}^e_{\theta^H}(p, \rho) \geq (R - \rho_0) I_H, \quad \bar{c}^e_{\theta^L}(p, \rho) \geq (R - \rho_0) I_L$$

$$\begin{aligned} \underline{c}^e_{\theta^H}(p, \rho) &\geq 0, \quad \underline{c}^e_{\theta^L}(p, \rho) \geq 0 \\ c^e_{\theta^H, L}(p, \rho) &\geq 0, \quad c^e_{\theta^L, L}(p, \rho) \geq 0 \end{aligned}$$

3.7.2 Proof of Lemma 8

Consider the maximization problem:

$$\max_{\{\chi_{\theta^H}, \chi_{\theta^L}, I_H, I_L\}} S\left(\chi_{\theta^H}, \chi_{\theta^L}, I_H, I_L\right)$$

subject to

$$
\begin{aligned}
& S\left(\chi_{\theta^H},\chi_{\theta^L},I_H,I_L\right)-\alpha\mathbb{E}^{\theta^H}\left[\mathbf{c}^e_{\theta^H}\chi_{\theta^H}\right]-(1-\alpha)\,\mathbb{E}^{\theta^L}\left[\mathbf{c}^e_{\theta^L}\chi_{\theta^L}\right]-\alpha I_H-(1-\alpha)\,I_L+A \\
- \quad & \alpha I_H\int\int\rho\chi_{\theta^H}(p,\rho)\,d\Phi_P\left(p|\theta^H,\rho\right)dF_{\theta^H}(\rho)-(1-\alpha)\,I_L\int\int\rho\chi_{\theta^L}(p,\rho)\,d\Phi_P\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)\geq 0
\end{aligned}
$$

$$
\mathbb{E}^{\theta^L}\left[\mathbf{c}^e_{\theta^L}\chi_{\theta^L}\right]\geq\mathbb{E}^{\theta^L}\left[\mathbf{c}^e_{\theta^H}\chi_{\theta^H}\right]
$$

Both constraints will hold with equality, thus I can solve the first constraint for I_H and the second constraint for I_L. After subsituting in the optimal payments $\mathbf{c}^e$ and rearranging, the maximization problem can be rewritten as:

$$
V^*=\max_{\{\chi^H,\chi^L\}}A\frac{\tilde{S}\left(\chi_{\theta^H},\chi_{\theta^L}\right)}{\tilde{D}\left(\chi_{\theta^H},\chi_{\theta^L}\right)}
$$

where

$$
\begin{aligned}
\tilde{S}\left(\chi_{\theta^H},\chi_{\theta^L}\right) \equiv\; & \alpha l\left(\theta^H\right)+\alpha\left(\theta^H R-l\left(\theta^H\right)\right)\int\int\chi_{\theta^H}(p,\rho)\,d\Phi_P\left(p|\theta^H,\rho\right)dF_{\theta^H}(\rho) \\
& +(1-\alpha)\,l\left(\theta^L\right)+(1-\alpha)\left(\theta^L R-l\left(\theta^L\right)\right)\int\int\chi_{\theta^H}(p,\rho)\,d\Phi_P\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)
\end{aligned}
$$

and

$$
\begin{aligned}
\tilde{D}\left(\chi_{\theta^H},\chi_{\theta^L}\right) \equiv\; & \alpha\left(1-l\left(\theta^H\right)+\int\int\left(\rho-\theta^H\rho_0+l\left(\theta^H\right)\right)\chi_{\theta^H}(p,\rho)\,d\Phi_P\left(p|\theta^H,\rho\right)dF_{\theta^H}(\rho)\right) \\
& +(1-\alpha)\frac{\int\int\chi_{\theta^H}(p,\rho)\,d\Phi_P\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)}{\int\int\chi_{\theta^L}(p,\rho)\,d\Phi_P\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)} \\
& \times\left(1-l\left(\theta^L\right)+\int\int\left(\rho-\theta^L\rho_0+l\left(\theta^L\right)\right)\chi_{\theta^L}(p,\rho)\,d\Phi_P\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)\right)
\end{aligned}
$$

I conjecture that $\chi_\theta(p,\rho)$ takes the form

$$
\chi_\theta(p,\rho)=\begin{cases}1 & \text{if } p\geq\bar{p}_\theta(\rho)\\ 0 & \text{otherwise}\end{cases}
$$

for some thresholds $\bar{p}_\theta(\rho)$. I will show that this conjecture is correct under a condition on the parameters of the model. As discussed in the main text, for given threshold $\bar{p}_\theta(\rho)$, the endogenous distribution of the price p is given by:

$$
\phi_P\left(p|\theta,\rho;\bar{p}_\theta(\rho)\right)=\begin{cases}\phi(R\theta-p) & \text{if } p\geq R\theta+\bar{p}_\theta(\rho)-l(\theta)\\ \gamma\phi\left(R\theta^L-p\right) & \text{if } \bar{p}_\theta(\rho)\leq p<R\theta+\bar{p}_\theta(\rho)-l(\theta)\\ (1-\gamma)\,\phi(l(\theta)-p) & \text{if } \bar{p}_\theta(\rho)-R\theta+l(\theta)\leq p<\bar{p}_\theta(\rho)\\ \phi(l(\theta)-p) & \text{if } p<\bar{p}_\theta(\rho)-R\theta+l(\theta)\end{cases}
$$

The maximization problem now becomes:

$$V^{*}=\max_{\{\bar{p}_{\theta^H}(\rho),\bar{p}_{\theta^L}(\rho)\}} A\frac{\tilde{S}\left(\bar{p}_{\theta^H}(\rho),\bar{p}_{\theta^L}(\rho)\right)}{\tilde{D}\left(\bar{p}_{\theta^H}(\rho),\bar{p}_{\theta^L}(\rho)\right)}$$

where

$$\begin{aligned}\tilde{S}\left(\bar{p}_{\theta^H}(\rho),\bar{p}_{\theta^L}(\rho)\right) \equiv\ & \alpha l\left(\theta^H\right)+\alpha\left(\theta^H R-l\left(\theta^H\right)\right)\int\int_{\bar{p}_{\theta^H}(\rho)}^{\infty} d\Phi_P\left(p|\theta^H,\rho\right)dF_{\theta^H}(\rho)\\ & +(1-\alpha)\,l\left(\theta^L\right)+(1-\alpha)\left(\theta^L R-l\left(\theta^L\right)\right)\int\int_{\bar{p}_{\theta^H}(\rho)}^{\infty} d\Phi_P\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)\end{aligned}$$

and

$$\begin{aligned}\tilde{D}\left(\bar{p}_{\theta^H}(\rho),\bar{p}_{\theta^L}(\rho)\right) \equiv\ & \alpha\left(1-l\left(\theta^H\right)+\int\int_{\bar{p}_{\theta^H}(\rho)}^{\infty}\left(\rho-\theta^H\rho_0+l\left(\theta^H\right)\right)d\Phi_P\left(p|\theta^H,\rho\right)dF_{\theta^H}(\rho)\right)\\ & +(1-\alpha)\frac{\int\int_{\bar{p}_{\theta^H}(\rho)}^{\infty} d\Phi_P\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)}{\int\int_{\bar{p}_{\theta^L}(\rho)}^{\infty} d\Phi_P\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)}\\ & \times\left(1-l\left(\theta^L\right)+\int\int_{\bar{p}_{\theta^L}(\rho)}^{\infty}\left(\rho-\theta^L\rho_0+l\left(\theta^L\right)\right)d\Phi_P\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)\right)\end{aligned}$$

Note maximizing w.r.t. $\bar{p}_{\theta^L}(\rho)$ is equivalent to solving

$$\max_{\bar{p}_{\theta^L}(\rho)}\frac{\left(\theta^L R-l\left(\theta^L\right)\right)\int\int_{\bar{p}_{\theta^L}(\rho)}^{\infty} d\Phi_P\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)}{1-l\left(\theta^L\right)+\int\int_{\bar{p}_{\theta^L}(\rho)}^{\infty}\left(\rho-\theta^L\rho_0+l\left(\theta^L\right)\right)d\Phi_P\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)} \tag{3.12}$$

The first order condition of (3.12) is exactly the same as in the symmetric information benchmark. Thus, the optimal χ_{θ^L} is undistorted and does not depend on p. In particular, there exists a constant threshold ρ_L^* such that

$$\chi_{\theta^L}(\rho)=\begin{cases}1 & \text{if } \rho\leq\rho_L^*\\ 0 & \text{otherwise}\end{cases}$$

Denote with V_L^* the optimal value of (3.12) and plug the optmal continuation rule $\chi_{\theta^L}(\rho)$ into (??) to obtain (with a slight abuse of notation):

$$V^{*}=\max_{\bar{p}_{\theta^H}(\rho)} A\frac{\tilde{S}\left(\bar{p}_{\theta^H}(\rho),\rho_L^*\right)}{\tilde{D}\left(\bar{p}_{\theta^H}(\rho),\rho_L^*\right)} \tag{3.13}$$

Consider now the integral:

$$
\begin{aligned}
& \int \int_{\bar{p}_{\theta}(\rho)}^{\infty} d\Phi_P\left(p|\theta,\rho\right) dF_{\theta}\left(\rho\right) \\
= & \int \int_{R\theta+\bar{p}_{\theta^H}(\rho)-l(\theta)}^{\infty} \phi\left(R\theta - p\right) f_{\theta}\left(\rho\right) dp d\rho + \int \int_{\bar{p}_{\theta^H}(\rho)}^{R\theta+\bar{p}_{\theta^H}(\rho)-l(\theta)} \gamma\phi\left(R\theta - p\right) f_{\theta}\left(\rho\right) dp d\rho
\end{aligned}
$$

where I have used the expression of the equilibrium distribution $\Phi_P\left(p|\theta,\rho\right)$. We can differentiate it w.r.t. $\bar{p}_{\theta^H}\left(\rho\right)$ to obtain:

$$
-\gamma\phi\left(R\theta - \bar{p}_{\theta^H}\left(\rho\right)\right) f_{\theta}\left(\rho\right) - \left(1-\gamma\right)\phi\left(l\left(\theta\right) - \bar{p}_{\theta^H}\left(\rho\right)\right) f_{\theta}\left(\rho\right) \equiv -g_P^{\theta}\left(\bar{p}_{\theta^H}\left(\rho\right)\right) f_{\theta}\left(\rho\right)
$$

The FOC of (3.13) is then given by:

$$
\begin{aligned}
& -\alpha\left(\theta^H R - l\left(\theta^H\right)\right) g^H\left(\bar{p}_{\theta^H}\left(\rho\right)\right) f_{\theta^H}\left(\rho\right) - \left(1-\alpha\right)\left(\theta^L R - l\left(\theta^L\right)\right) g^L\left(\bar{p}_{\theta^H}\left(\rho\right)\right) f_{\theta^L}\left(\rho\right) \\
+ \quad & V^*\left(\alpha\left(\rho - \theta^H\rho_0 + l\left(\theta^H\right)\right) g^H\left(\bar{p}_{\theta^H}\left(\rho\right)\right) f_{\theta^H}\left(\rho\right) + \left(1-\alpha\right)\frac{\theta^L R - l\left(\theta^L\right)}{V_L^*} g^L\left(\bar{p}_{\theta^H}\left(\rho\right)\right) f_{\theta^L}\left(\rho\right)\right) = 0
\end{aligned}
$$

or, collecting the terms containing $\bar{p}^*\left(\rho\right)$,

$$
\frac{g^L\left(\bar{p}_{\theta^H}\left(\rho\right)\right) f_{\theta^L}\left(\rho\right)}{g^H\left(\bar{p}_{\theta^H}\left(\rho\right)\right) f_{\theta^H}\left(\rho\right)} = \frac{\alpha}{1-\alpha} \frac{\theta^H R - l\left(\theta^H\right) - V^*\left(\rho - \theta^H\rho_0 + l\left(\theta^H\right)\right)}{\left(\theta^L R - l\left(\theta^L\right)\right)\left(\frac{V^*}{V_L^*} - 1\right)}
$$

which is (3.6) in the main text. Notice that the first order condition depends on the value of the problem V^* and on the value for the low type V_L^*.

For given value of V_L^* and V^* let $\bar{p}_{\theta^H}\left(\rho; V^*, V_L^*\right)$ be the solution to the equation (where I am explicitly highlghting the dependence on V^* and V_L^*)

$$
\frac{g^L\left(\bar{p}_{\theta^H}\left(\rho; V^*, V_L^*\right)\right) f_{\theta^L}\left(\rho\right)}{g^H\left(\bar{p}_{\theta^H}\left(\rho; V^*, V_L^*\right)\right) f_{\theta^H}\left(\rho\right)} = \frac{\alpha}{1-\alpha} \frac{\theta^H R - l\left(\theta^H\right) - V^*\left(\rho - \theta^H\rho_0 + l\left(\theta^H\right)\right)}{\left(\theta^L R - l\left(\theta^L\right)\right)\left(\frac{V^*}{V_L^*} - 1\right)} \tag{3.14}
$$

First notice that the value of V_L^* can be found by maximizing (3.12) regardless of the threshold $\bar{p}_{\theta^H}\left(\rho; V^*, V_L^*\right)$. Also, given V_L^*, for each value of V^* (3.14) can be solved to find the threshold $\bar{p}_{\theta^H}\left(\rho; V^*, V_L^*\right)$ and hence a continuation rule $\chi_{\theta^H}\left(p, \rho; V^*, V_L^*\right)$.

Let

$$
\Gamma\left(V^*, V_L^*\right) = A\frac{\tilde{S}\left(\bar{p}_{\theta^H}\left(\rho; V^*, V_L^*\right), \rho_L^*\right)}{\tilde{D}\left(\bar{p}_{\theta^H}\left(\rho; V^*, V_L^*\right), \rho_L^*\right)}
$$

be the value of the objective function in 3.13 at the solution $\bar{p}_{\theta^H}(\rho; V^*, V_L^*)$, ρ_L^*, and V_L^*. The value V^* of 3.13 is then given by the solution to the equation

$$V^* = \Gamma(V^*, V_L^*)$$

I solve this problem numerically.

From the zero-profit condition of the investors and the IC constraint we can get the expressions for the initial levels of investment:

$$I_H = \frac{A}{\tilde{D}(\chi_{\theta^H}, \chi_{\theta^L})} \tag{3.15}$$

and

$$I_L = \frac{\int\int \chi_{\theta^H}(p,\rho)\, d\Phi_P(p|\theta^L,\rho)\, dF_{\theta^L}(\rho)}{\int\int \chi_{\theta^L}(p,\rho)\, d\Phi_P(p|\theta^L,\rho)\, dF_{\theta^L}(\rho)} \frac{A}{\tilde{D}(\chi_{\theta^H}, \chi_{\theta^L})} \tag{3.16}$$

Finally, we need to verify that original conjecture that $\chi_\theta(p,\rho)$ can be represented as in (3.4) is true. It turns out that the conjecture is true provided that the parameters of the model R, θ^H, θ^L, $l(\theta^H)$, $l(\theta^L)$ satisfy

$$\frac{d}{dp}\left(\frac{g^L(p)}{g^H(p)}\right)\Bigg|_{p=\left(R\theta^L+l(\theta^H)\right)/2} < 0$$

This restriction is necessary and sufficient for the likelihood ratio $g^L(\bar{p}_{\theta^H}(\rho))/g^H(\bar{p}_{\theta^H}(\rho))$ in (3.6) to be monotone (decreasing) in the threshold $\bar{p}_{\theta^H}(\rho)$.

3.7.3 Proof of Proposition 21

From (3.5), for given ρ, the probability of being saved is

$$\Pr(p \geq \bar{p}_{\theta^H}(\rho)\,|\rho) = \gamma \int_{\bar{p}_{\theta^H}(\rho)}^{R\theta+\bar{p}_{\theta^H}(\rho)-l(\theta)} \phi(R\theta - p)\, dp + \int_{R\theta+\bar{p}_{\theta^H}(\rho)-l(\theta)}^{\infty} \phi(R\theta - p)\, dp$$

Let $u = R\theta^H - p$, then

$$\begin{aligned} &= \gamma \int_{l(\theta^H)-\bar{p}_{\theta^H}(\rho)}^{R\theta-\bar{p}_{\theta^H}(\rho)} \phi(u)\, du + \int_{-\infty}^{l(\theta)-\bar{p}_{\theta^H}(\rho)} \phi(u)\, dp \\ &= \gamma\Phi\left(R\theta^H - \bar{p}_{\theta^H}(\rho)\right) + (1-\gamma)\,\Phi\left(l(\theta^H) - \bar{p}_{\theta^H}(\rho)\right) \end{aligned}$$

which can be differentiated w.r.t. σ_u^2 holding $\bar{p}_\theta(\rho)$ constant:

$$-\frac{1}{2}\frac{1}{\sigma_u^2}\left(\gamma\int_{l(\theta^H)-\bar{p}_{\theta^H}(\rho)}^{R\theta-\bar{p}_{\theta^H}(\rho)}\frac{1}{\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}du+\int_{-\infty}^{l(\theta)-\bar{p}_{\theta^H}(\rho)}\frac{1}{\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}dp\right)$$
$$+\left(\gamma\int_{l(\theta^H)-\bar{p}_{\theta^H}(\rho)}^{R\theta-\bar{p}_{\theta^H}(\rho)}\frac{u^2}{2\left(\sigma_u^2\right)^2\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}du+\int_{-\infty}^{l(\theta)-\bar{p}_{\theta^H}(\rho)}\frac{u^2}{2\left(\sigma_u^2\right)^2\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}dp\right)$$

$$=\frac{1}{2\left(\sigma_u^2\right)^2}\left(\gamma\int_{l(\theta^H)-\bar{p}_{\theta^H}(\rho)}^{R\theta-\bar{p}_{\theta^H}(\rho)}\frac{u^2-\sigma_u^2}{\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}du+\int_{-\infty}^{l(\theta)-\bar{p}_{\theta^H}(\rho)}\frac{u^2-\sigma_u^2}{\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}dp\right)$$
$$=\frac{1}{2\left(\sigma_u^2\right)^2}\left(\gamma\int_{-\infty}^{R\theta-\bar{p}_{\theta}(\rho)}\frac{u^2-\sigma_u^2}{\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}du+(1-\gamma)\int_{-\infty}^{l(\theta)-\bar{p}_{\theta^H}(\rho)}\frac{u^2-\sigma_u^2}{\sqrt{2\pi\sigma_u^2}}\exp\left\{-\frac{u^2}{2\sigma_u^2}\right\}dp\right)$$

$$\begin{aligned}
\text{if } R\theta-\bar{p}_\theta(\rho) &< 0 \text{ then derivative is positive}\\
\text{if } l(\theta)-\bar{p}_\theta(\rho) &> 0 \text{ then derivative is negative}\\
\text{if } R\theta-\bar{p}_\theta(\rho) &> 0>l(\theta)-\bar{p}_\theta(\rho) \text{ then derivative depends on } \gamma
\end{aligned}$$

3.7.4 Proof of Proposition 22

The proofs of (1) and (2) are analogous to the proofs of the corresponding results in Propositions 18, 19, and 20.

From the zero-profit condition in P4 we can solve for the initial investment level of the two types of projects:

$$I_\theta^{AS}=\frac{A}{1-l(\theta)+\int\int\left(\rho-\theta\rho_0+l(\theta)\right)\chi_\theta^{AS}(p,\rho)\,d\Phi_P^{AS}(p|\theta,\rho)\,dF_\theta(\rho)}$$

This proves (3).

The maximization problem can then be rewritten as:

$$\max_{\left\{\chi_{\theta^H}^{AS},\chi_{\theta^L}^{AS}\right\}}\left(R\theta^H\int\int\chi_{\theta^H}^{AS}(p,\rho)\,d\Phi_P^{AS}\left(p|\theta^H,\rho\right)dF_{\theta^H}(\rho)-1-\int\int\rho\chi_{\theta^H}^{AS}(p,\rho)\,d\Phi_P^{AS}\left(p|\theta^H,\rho\right)dF_{\theta^H}(\rho)\right)I_H$$

subject to the IC constraint for the low type:

$$\begin{aligned}
&\left(R\theta^L\int\int\chi_{\theta^L}^{AS}(p,\rho)\,d\Phi_P^{AS}\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)-1-\int\int\rho\chi_{\theta^L}^{AS}(p,\rho)\,d\Phi_P^{AS}\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)\right)I_{\theta^L}^{AS}\\
\geq\ &\left(R\theta^L\int\int\chi_{\theta^H}^{AS}(p,\rho)\,d\Phi_P^{AS}\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)-1-\int\int\rho\chi_{\theta^H}^{AS}(p,\rho)\,d\Phi_P^{AS}\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)\right)I_H
\end{aligned}$$

As usual, I conjecture that $\chi_\theta^{AS}(p,\rho)$ takes the form

$$\chi_{\theta}^{AS}(p,\rho)=\begin{cases}1 & \text{if } p\geq \bar{p}_{\theta}^{AS}(\rho)\\ 0 & \text{otherwise}\end{cases}$$

for some thresholds $\bar{p}_{\theta}^{AS}(\rho)$. In turn, this implies that the equilibrium distribution for the asset price is identical to (3.5) in the main text:

$$\phi_{P}^{AS}(p|\theta,\rho)=\begin{cases}\phi(R\theta-p) & \text{if } p\geq R\theta+\bar{p}_{\theta}^{AS}(\rho)-l(\theta)\\ \gamma\phi(R\theta^{L}-p) & \text{if } \bar{p}_{\theta}^{AS}(\rho)\leq p< R\theta+\bar{p}_{\theta}^{AS}(\rho)-l(\theta)\\ (1-\gamma)\phi(l(\theta)-p) & \text{if } \bar{p}_{\theta}^{AS}(\rho)-R\theta+l(\theta)\leq p<\bar{p}_{\theta}^{AS}(\rho)\\ \phi(l(\theta)-p) & \text{if } p<\bar{p}_{\theta}^{AS}(\rho)-R\theta+l(\theta)\end{cases}\tag{3.17}$$

I can then rewrite the problem as:

$$\max_{\left\{\bar{p}_{\theta H}^{AS},\,\bar{p}_{\theta L}^{AS}\right\}}\left(R\theta^{H}\int_{0}^{\infty}\int_{\bar{p}_{\theta H}^{AS}(\rho)}^{\infty}d\Phi_{P}^{AS}(p|\theta^{H},\rho)\,dF_{\theta^{H}}(\rho)-1-\int_{0}^{\infty}\int_{\bar{p}_{\theta H}^{AS}(\rho)}^{\infty}\rho d\Phi_{P}^{AS}(p|\theta^{H},\rho)\,dF_{\theta^{H}}(\rho)\right)I_{H}$$

subject to the IC constraint for the low type:

$$\begin{aligned}&\left(R\theta^{L}\int_{0}^{\infty}\int_{\bar{p}_{\theta L}^{AS}(\rho)}^{\infty}d\Phi_{P}^{AS}(p|\theta^{L},\rho)\,dF_{\theta^{L}}(\rho)-1-\int_{0}^{\infty}\int_{\bar{p}_{\theta L}^{AS}(\rho)}^{\infty}\rho d\Phi_{P}^{AS}(p|\theta^{L},\rho)\,dF_{\theta^{L}}(\rho)\right)I_{\theta^{L}}^{AS}\\ \geq\ &\left(R\theta^{L}\int_{0}^{\infty}\int_{\bar{p}_{\theta H}^{AS}(\rho)}^{\infty}d\Phi_{P}^{AS}(p|\theta^{L},\rho)\,dF_{\theta^{L}}(\rho)-1-\int_{0}^{\infty}\int_{\bar{p}_{\theta H}^{AS}(\rho)}^{\infty}\rho d\Phi_{P}^{AS}(p|\theta^{L},\rho)\,dF_{\theta^{L}}(\rho)\right)I_{H}\end{aligned}$$

Given that the IC constraint will hold with equality at the optimal solution, we can use it to substitute for $I_{\theta^{H}}^{AS}$ in the objective function.

Let V_{H}^{AS} and V_{L}^{AS} be the expected utility that the high and low type get in equilibrium, respectively. The continuation rule for the low type is once again undistorted. In fact, the first order condition w.r.t. $\bar{p}_{\theta L}^{AS}(\rho)$ is:

$$\left(1+V_{L}^{AS}\right)\rho-R\theta^{L}-V_{L}^{AS}\left(\theta\rho_{0}+l(\theta)\right)\geq 0\tag{3.18}$$

which is independent of $\bar{p}_{\theta L}^{AS}(\rho)$. Also in the case of adverse selection, therefore, the continuation rule for the low type has the form:

$$\chi_{\theta^{L}}(\rho)=\begin{cases}1 & \text{if } \rho\leq\bar{\rho}_{L}^{AS}\\ 0 & \text{otherwise}\end{cases}$$

where $\bar{\rho}_{L}^{AS}$ is the solution to 3.18. This proves (4).

To find the optimal continuation rule for the high type, we can use the IC constraint to sustitute

$I^{AS}_{\theta^H}$ into the objective function:

$$\max_{\left\{\chi^{AS}_{\theta^H},\chi^{AS}_{\theta^L}\right\}}\left(R\theta^H\int\int\chi^{AS}_{\theta^H}(p,\rho)\,d\Phi^{AS}_P\left(p|\theta^H,\rho\right)dF_{\theta^H}(\rho)-1-\int\int\rho\chi^{AS}_{\theta^H}(p,\rho)\,d\Phi^{AS}_P\left(p|\theta^H,\rho\right)dF_{\theta^H}(\rho)\right)I^{AS}_{\theta^H}$$

$$V^{AS}_H\equiv\max_{\bar{p}^{AS}_{\theta^H}(\rho)}\frac{R\theta^H\int_0^\infty\int_{\bar{p}^{AS}_{\theta^H}(\rho)}^\infty d\Phi^{AS}_P\left(p|\theta^H,\rho\right)dF_{\theta^H}(\rho)-1-\int_0^\infty\int_{\bar{p}^{AS}_{\theta^H}(\rho)}^\infty\rho\, d\Phi^{AS}_P\left(p|\theta^H,\rho\right)dF_{\theta^H}(\rho)}{R\theta^L\int_0^\infty\int_{\bar{p}^{AS}_{\theta^H}(\rho)}^\infty d\Phi^{AS}_P\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)-1-\int_0^\infty\int_{\bar{p}^{AS}_{\theta^H}(\rho)}^\infty\rho\, d\Phi^{AS}_P\left(p|\theta^L,\rho\right)dF_{\theta^L}(\rho)}V^{AS}_L$$

where I have evaluated the IC constraint at the optimal continuation rule of the low type (this is the reason V^{AS}_L is in the objective function). The first order condition w.r.t. $\bar{p}^{AS}_{\theta^H}(\rho)$ gives:

$$\frac{g^{AS}_P\left(\bar{p}^{AS}_{\theta^H}(\rho)|\theta^L,\rho\right)}{g^{AS}_P\left(\bar{p}^{AS}_{\theta^H}(\rho)|\theta^H,\rho\right)}\frac{f_{\theta^L}(\rho)}{f_{\theta^H}(\rho)}=\frac{V^{AS}_L\left(\rho-R\theta^H\right)}{V^{AS}_H\left(R\theta^L-\rho\right)} \tag{3.19}$$

where

$$g^{AS}_P\left(\bar{p}^{AS}_{\theta^H}(\rho)|\theta,\rho\right)=\gamma\phi\left(R\theta-\bar{p}^{AS}_{\theta^H}(\rho)\right)+(1-\gamma)\phi\left(l(\theta)-\bar{p}^{AS}_{\theta^H}(\rho)\right)$$

3.19 implicitly defines $\bar{p}^{AS}_{\theta^H}(\rho)$. Of couse, as we did also in Proposition 20, solving 3.19 for a given value of V^{AS}_H is not enough since V^{AS}_H itself is a function of the continuation rules. Instead, V^{AS}_H has to satisfies a fixed point problem equivalent to the one shown in the proof of Proposition 20. This proves (5)

3.8 Appendix 2. Extension: Adverse selection

In the main text I have assumed that when investors and entrepreneurs meet they do so under symmetric information. In particular, entrepreneurs learn their types only after they have signed for a menu of contracts with the investors. In section 3.2 I have explained that the main reason for making this assumption is that it makes the analysis easier without altering the main conclusions. In this appendix, I consider the alternative assumption of having entrepreneurs learn their types before the contracting stage.

There are two important reasons for writing this appendix. First, I can prove that the results in the main text are not specific to the particular way investors and entrepreneurs interact, nor are they specific to the definition of equilibrium adopted. On the contrary, when the equilibrim exists, I can prove that the main insights go through with few interesting differences. One important difference is that, in the equilibrium I study here, there is no cross-subsidization between different types of projects. In other words, investors offer contracts that make zero expected profits separately for each type. On the contrary, under the other assumption, investors were making zero profits only on average between the two types.

As shown in Rothschild and Stiglitz (1976), if a firm offered a contract which cross-subsidizes between different types of agents, an investor would find it profitable to offer another contract which "cream skims" the high types away from the cross-subsidizing firm. This, in turn, would make the cross-subsidizing contract unprofitable. This feature of the equilibrium is particularly interesting for my model. I show that, when cross-subsidization is absent, any *ex-ante* information – that is, any information that arrives before the contract is signed – will not be included in the contract. On the contrary, the *ex-post* asset prices considered in this model will still be part of the optimal contract.

Different equilibrium concepts have been considered in the literature of competition with adverse selection, but so far none of them has proven to be conclusive.

Consider, for example, the seminal contribution of Rothschild and Stiglitz (1976). They look for a Nash equilibrium of the game where firms compete by offering insurance contracts to risk averse agents. They prove that, when such equilibrium exists, firms separate the different types of agents by selling them different contracts. Moreover, as already discussed above, in equilibrium firms do not cross-subsidize among differet types of agents. The well known problem with this equilibrium concept is that it fails to exist for a non trivial subset of parameters.

Other papers have tried to get around this problem by introducing different definitions of equilibrium. Riley (1979), ?, and ? have extended the game so that competitors can react when a firm deviates from the equilibrium. This solves the problem of existence in Rothschild and Stiglitz (1976), but the definition of equilibrium is not totally appealing. ? extends the game to a dynamic setting and shows that the results of Rothschild and Stiglitz (1976) are not entirely robust to this modification of the game.

Prescott and Townsend (1984) study the properties of Walrasian equilibria in economies with different forms of asymmetric information. Unlike the above game-theoretic literature, in a Walrasian equilibrium there is a market where price-taker firms offer insurance contracts to risk averse, price-taker individuals. Prices then adjust to clear all the markets. The key point is that in general different types of agents face different prices (insurance for a high risk individual is likely to be more expensive). In equilibrium each agent has to be willing to purchase the insurance contract from the market of his type. Prescott and Townsend (1984) show that a Walrasian equilibrium always exists but, in the presence of adverse selection, the equilibrium may fail to be constrained efficient. In particular, in the simple Rothschild-Stiglitz economy, they show that the equilibrium fails to be constrained efficient for the same parameters for which the Nash equilibrium used by Rothschild and Stiglitz (1976) fails to exists.

In a more recent paper, Bisin and Gottardi (2006) show that the Walrasian equilibrium in Prescott and Townsend (1984) is not efficient because of an externality that each type of agents exerts on the other types when buying an insurance contract. Also, they characterize the Walrasian equilibrium for the Rotschild - Stiglitz economy and show that it is always given by the separating contract with no cross-subsidization among different types of agents. The externality works through the incentive compatibility constraint. Bisin and Gottardi (2006) deal with this externality by considering Lindhal equilibria. This equilibrium concept, together with an exogenous restriction on the types of contracts that agents can buy, delivers existence and efficieny of the equilibrium for the Rotschild - Stiglitz economy. However, a recent paper by Rustichini and Siconolfi (2008) shows that, if the exogenous restriction on the set of contracts in Bisin and Gottardi (2006) is removed, the equilibrium may again fail to exist even if a Lindhal equilibrium is considered.

Given the absence of a conclusive notion of equilibrium for economies with adverse selection, I adopt a definition similar to that in Rothschild and Stiglitz (1976). I then show that the equilibrium is separating and there is no cross-subsidization between types.

Also, let me point out that the absence of cross-subsidization is not a feature of some unusual notion of equilibrium. On the contrary, no cross-subsidization happens both in the equilibrium of Rothschild and Stiglitz (1976), when the latter exists, and in the Walrasian equilibrium of Prescott and Townsend (1984), which always exists, but sometimes is (constrained) inefficient. Thus, under the informational assumptions of this section, an equilibrium with no cross-subsidization between types is indeed reasonable.

The fact that in equilibriumm there is no cross-subsidization has an effect on how information is used in the contract. The fact that a different definition of equilibrium affects the way agents use information should not be a surprise. However, I show that the absence of cross-subsidization implies that if an investor has *ex-ante* (that is, before the contracting stage) information about the type of the entrepreneur, he will not use it in the optimal contract. On the contrary, in equilibrium investors will find it optimal to condition the contract only to *ex-post* information – like the asset price I consider in this model.

To see why this is the case, remember that in equilibrium investors have to break-even for each type separately. Now, imagine to provide investors with some information about the type of the entrepreneurs before the contract is signed. For concreteness, suppose that when an investor meets an entrepreneur, he can observe the entrepreneur's past performance which is correlated with the type of the current project. Will the investor use this information to "skim" the good types? In an equilibrium with no cross-subsidization the answer is no. To see this, say that the outcome of the previous project can be either success of failure. Now, imagine that the investor decides to offer contracts only to the entrepreneurs who were successful in the previous project. When there is no cross-subsidization between types, the equilibrium contracts are insensitive to the fraction of high types in the pool. Thus, an investor who decides to focus only on the group of previously successful entrepreneurs will face the same basic incentive constraints within this group as an investor who does not condition on the outcome of the previous project. Hence, there is no reason in equilibrium to use the information provided by the outcome of the previous project.

This result is analogous to those obtained in the literature on categorical discrimination in insurance markets. In particular, Crocker and Snow (1986) and, more recently, Rothschild (2011) are both concerned with whether banning categorization in the insurance market can improve the equilibrium efficiency. Interestingly, Rothschild (2011) studies the efficiency of banning categorization across different equilibrium concepts. He shows that, when there is no cross-subsidization in equilibrium, then categorization based on risk-related characteristics does increase equilibrium welfare. On the other hand, when other definitions of equilibrium with cross-subsidization are considered, welfare does increase when categorical discrimination is allowed[4].

There is then a stark contrast between *ex-ante* information contained, for example, in entrepreneurs' past performances or in past market signals and the *ex-post* information provided by the financial markets considered in this paper. In fact, since investors do not cross-subsidize in equilibrium and use market information only insofar as it relaxes the incentive constraint of the entrepreneurs, only ex-post information matters and is incorporated in the contract.

As said above, for simplicity, the equilibrium concept I adopt in this section is the standard Rothschild and Stiglitz (1976) equilibrium. It is well known that this equilibrium may fail to exist for some parameter values. Since the issue of existence of this equilibrium is not related to the question I am interested in this paper, I will not provide a proof of existence, nor conditions under which an equilibrium exists. In general, though, as it is the case in Rothschild and Stiglitz (1976), the equlibrium exists whenever the fraction of high types α is "high enough".

Formally, when the equilibrium of the game where investors offer contracts to informed entrepreneurs exists, it is the solution to the following problem:

[4] The equilibrium concepts considered are the Riley (1979a,b) "reactive" equilibrium (which coincides with the Rothschild-Stiglitz equilibrium when this exists), the Wilson (1977) equilibrium and the Miyazaki (1977)-Wilson (1977)-Spence (1978) "anticipatory" equilibria.

$$\max_{C \subseteq \mathbb{C}^{MH}} U\left(C_{\theta^H};\theta^H, \Phi_p\left(\cdot|\theta^H,\rho\right)\right) \tag{P4}$$

subject to the contract delivering zero profits to the investor who offers it:

$$\pi\left(C_\theta;\theta,\Phi_p\left(\cdot|\theta,\rho\right)\right)=0,\ \ \forall\theta\in\Theta$$

and that the contract is incentive compatible

$$U\left(C_{\theta^i};\theta^i,\Phi_p\left(\cdot|\theta^i,\rho\right)\right)\geq U\left(C_{\theta^j};\theta^i,\Phi_p\left(\cdot|\theta^i,\rho\right)\right),\quad i,j=L,H$$

Note that now the zero-profit condition applies type by type (this is the no cross-subsidization result). Also, in the separating equilibrium the high type entrepreneur receives the highest possible payoff compatible with the low type reporting his type truthfully. The following proposition characterizes the equilibrium allocations. I use the superscript "AS" (Adverse Selection) to identify the objects of this section.

Proposition 22. *The solution to P4 (which coincides with the equilibrium of the game when the latter exists) is given by a menu of contracts $\mathcal{C}^{AS}$ with the following properties:*

1. *The entrepreneur consumes the minimum share of output if the project is successful and nothing otherwise:*
$$\bar{c}_\theta^{e,AS}(p,\rho)=(R-\rho_0)\,I_\theta^{AS},\ \underline{c}_\theta^{e,AS}(p,\rho)=0,\ c_{\theta,L}^{e,AS}(p,\rho)=0$$

2. *The investor receives all the pleadgeble income if the project is successful and the liquidation value if the project is liquidated:*
$$\bar{c}_\theta^{i,AS}(p,\rho)=\rho_0 I_\theta^{AS},\ \underline{c}_\theta^{i,AS}(p,\rho)=0,\ c_{\theta,L}^{i,AS}(p,\rho)=l\,(\theta)\,I_\theta^{AS}$$

3. *The low type obtains the same liquidity insurance as in the symmetric information benchmark:*
$$\chi_{\theta^L}^{AS}(\rho)=\begin{cases}1 & \text{if } \rho\leq\bar{\rho}_L^{AS}\\ 0 & \text{otherwise}\end{cases}$$
where $\bar{\rho}_L^{AS}$ is the solution to 3.18 in the appendix.

4. *The continuation rule for the high type $\chi_{\theta^H}^{AS}(p,\rho)$ depends on the asset price. Formally, for each ρ there exists a threshold $\bar{p}_{\theta^H}^{AS}(\rho)$ such that*
$$\chi_{\theta^H}^{AS}(p,\rho)=\begin{cases}1 & \text{if } p\geq\bar{p}_{\theta^H}^{AS}(\rho)\\ 0 & \text{otherwise}\end{cases}$$

where $\bar{p}^{AS}_{\theta^H}(\rho)$ *is the solution to* 3.19 *in the appendix.*

5. *Investment is given by:*

$$I_\theta^{AS} = \frac{A}{1 - l(\theta) + \int\int(\rho - \theta\rho_0 + l(\theta))\,\chi_\theta^{AS}(p,\rho)\,d\Phi_P^{AS}(p|\theta,\rho)\,dF_\theta(\rho)}$$

6. *The optimal contract determines an equilibrium distribution for the asset price* $\Phi_P^{AS}(p|\theta,\rho)$ *given by* (3.17) *in the appendix.*

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